

High-dimensional modeling and forecasting for wind power generation

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(authors in alphabetical order)

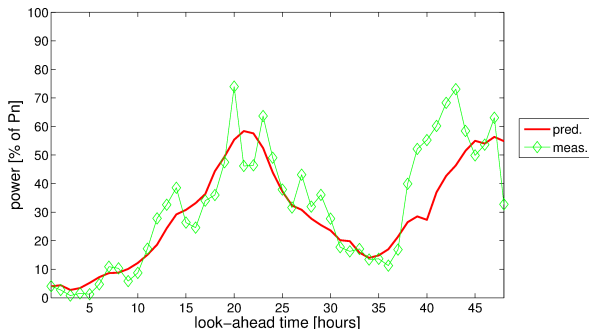
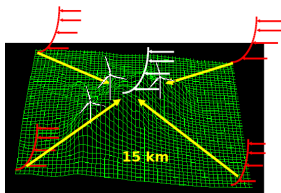
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YEQT Winter School on Energy Systems - 13 December 2017

- Motivations for high-dimension learning and forecasting
- General sparsity control for VAR models
- Online sparse and adaptive learning for VAR models
- Distributed learning
- Outlook

- 1 From single wind farms to entire regions (1000s)

The wind power forecasting problem is defined for a single location...

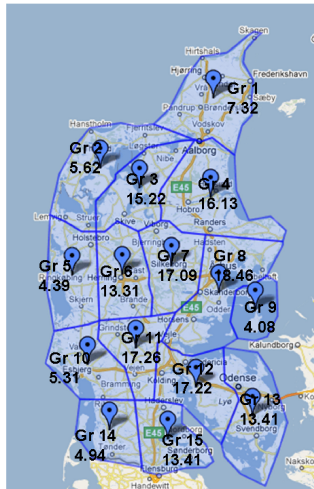


... or, if several locations, by considering each of them individually

(Note that, for simplicity, we will only look at very short-term forecasting in this talk, i.e., from a few mins to 1-hour ahead)

Many works showed that **forecast quality could be significantly improved:**

- by using data at offsite locations (i.e., other wind farms)
- based on spatio-temporal modelling (and the likes)

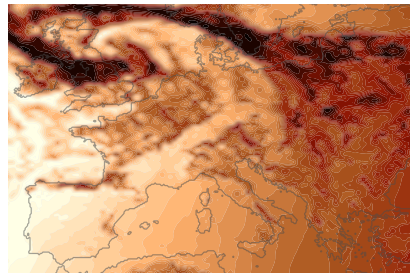
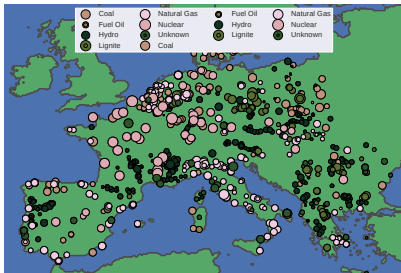
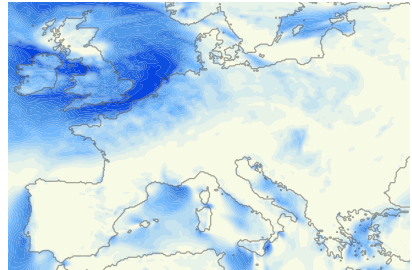


improvement of 1-hour ahead forecast RMSE

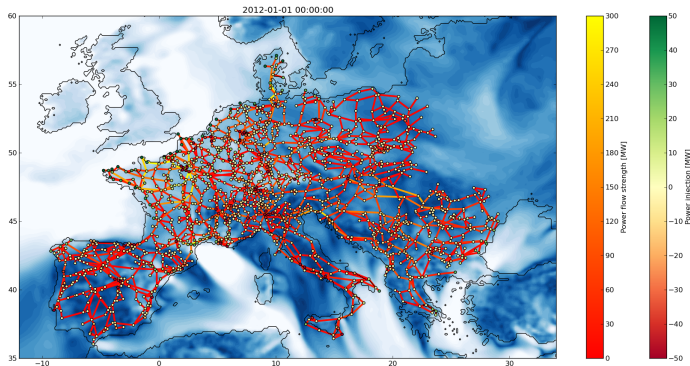
- A Danish example...
- Accounting for spatio-temporal effects allows for the correction of aggregated power forecasts **for horizons up to 8 hours ahead**
- Largest improvements at **horizons of 2-5 hours ahead**

Scaling it up

Ultimately, we would like to predict all wind power generation, also solar and load, at the scale of a continental power system, e.g. the European one



- The “**grand forecasting challenge**”: predict *renewable power generation*, *dynamic uncertainties* and *space-time dependencies* at once for the whole Europe...!



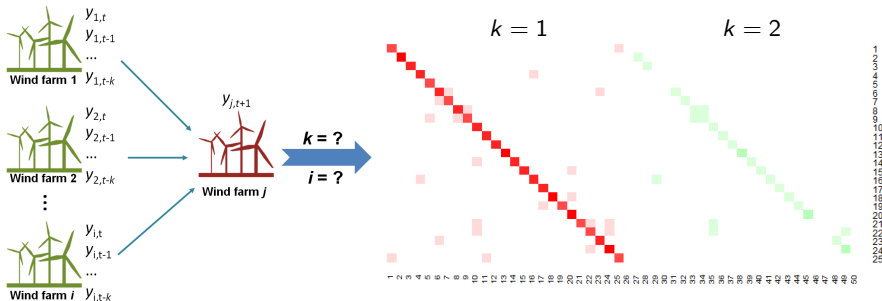
- **Linkage with future electricity markets:**

- Monitoring and forecasting of the complete “**Energy Weather**” over Europe
- Provides all necessary information for coupling of various existing markets (e.g., day-ahead, balancing), and deciding upon optimal cross-border exchanges

- ② A proposal for general sparsity control (not online though)

Traditional LASSO-VAR can **only provide overall sparse solutions**, but **not allow for fine-tuning different aspects of sparsity**, e.g. :

- overall number of nonzero coefficients of VAR (S_A), i.e. *the LASSO-VAR*
- number of explanatory wind farms used in VAR to explain target wind farm i (S_F^i)
- number of past observations of each explanatory wind farm to explain target wind farm i (S_P^i)
- number of nonzero coefficients to explain target wind farm i (S_N^i).



These aspects can be used to control the sparse structure of the solution as needed, especially **when prior knowledge on spatio-temporal characteristics of wind farms are available for sparsity-control and expected to improve the forecasting**.

How to freely control the sparse structure... [E. Carrizosa, et al. 2017]

- Introducing binary control variables γ_j^i and δ_{jk}^i
 - γ_j^i controls whether wind farm j is used to explain target wind farm i .
 - δ_{jk}^i controls whether the coefficient α_{jk}^i is zero or not.
- Reformulating the VAR estimation as a constrained mixed integer non-linear programming (MINLP) problem.

For example: $N = 3$ wind farms, VAR(2) with $p = 2$ lags

$$\begin{bmatrix} \gamma_1^1 & \gamma_2^1 & \gamma_3^1 \\ \gamma_1^2 & \gamma_2^2 & \gamma_3^2 \\ \gamma_1^3 & \gamma_2^3 & \gamma_3^3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \iff \mathbf{A} = \begin{bmatrix} \alpha_{11}^1 & 0 & \alpha_{31}^1 & \alpha_{12}^1 & 0 & \alpha_{32}^1 \\ 0 & \alpha_{21}^2 & 0 & 0 & \alpha_{22}^2 & 0 \\ \alpha_{11}^3 & 0 & \alpha_{31}^3 & \alpha_{12}^3 & 0 & \alpha_{32}^3 \end{bmatrix}$$

If additionally with control variable $\delta_{11}^3 = 0$, then

$$\mathbf{A} = \begin{bmatrix} \alpha_{11}^1 & 0 & \alpha_{31}^1 & \alpha_{12}^1 & 0 & \alpha_{32}^1 \\ 0 & \alpha_{21}^2 & 0 & 0 & \alpha_{22}^2 & 0 \\ 0 & 0 & \alpha_{31}^3 & \alpha_{12}^3 & 0 & \alpha_{32}^3 \end{bmatrix}$$

That is:

$$\gamma_j^i = 0 \iff \sum_{k=1}^p \delta_{jk}^i = 0 \qquad \delta_{jk}^i = 0 \iff \alpha_{jk}^i = 0$$

$$\min_{\alpha, \delta, \gamma} \sum_{i=1}^N \sum_{t=p}^T \left(y_{i,t+1} - \sum_{j=1}^N \sum_{k=1}^p \alpha_{jk}^i y_{j,t-k+1} \right)^2$$

$$\text{subject to } \delta_{jk}^i \leq \gamma_j^i, \forall k \in \mathbf{K}, i, j \in \mathbf{I}$$

$$\sum_{j=1}^N \gamma_j^i \leq S_F^i, \forall i \in \mathbf{I}$$

$$\sum_{k=1}^p \gamma_j^i \delta_{jk}^i \leq S_P^i, \forall i, j \in \mathbf{I}$$

$$\sum_{i=1}^N \sum_{j=1}^N \sum_{k=1}^p \delta_{jk}^i \leq S_A, \forall k \in \mathbf{K}, i, j \in \mathbf{I}$$

$$\sum_{j=1}^N \sum_{k=1}^p \delta_{jk}^i \leq S_N^i, \forall i \in \mathbf{I}$$

$$|\alpha_{jk}^i| \geq \eta_j^i \delta_{jk}^i, \forall k \in \mathbf{K}, i, j \in \mathbf{I}$$

$$\alpha_{jk}^i (1 - \delta_{jk}^i) = 0, \forall k \in \mathbf{K}, i, j \in \mathbf{I}$$

$$\delta_{jk}^i, \gamma_j^i \in \{0, 1\}, \forall k \in \mathbf{K}, i, j \in \mathbf{I}$$

- $\mathbf{I} = \{1, 2, \dots, N\}$
- $\mathbf{K} = \{1, 2, \dots, p\}$
- S_A - overall number of nonzero coefficients of VAR
- S_F^i - number of explanatory wind farms used in VAR to explain target wind farm i
- S_P^i - number of past observations of each explanatory wind farm to explain target wind farm i
- S_N^i - number of nonzero coefficients to explain target wind farm i
- η_j^i - a threshold requires that only coefficients with absolute value greater than or equal to η_j^i are effective otherwise will be zero.

Pros and cons of SC-VAR model

Pros

- allows for fully controlling the sparsity from different aspects.
- can be directly solved by off-the-shelf standard MINLP solvers.

Cons

- *SC-VAR allows for sparsity-control but doesn't tell how to control.* No information is available for setting so many parameters, which are practically intractable when dealing with high dimensional wind power forecasting.
- The constraint $\sum_{k=1}^P \gamma_j^i \delta_{jk}^i \leq S_P^i$ is nonlinear.
- The constraints are redundant: $S_F^i + S_P^i = S_N^i$, $\sum_{i \in I} S_N^i = S_A$
- The constraint $\sum \sum \sum \delta_{jk}^i \leq S_A$ makes the optimization problem non-decomposable, which slows down the computation.
- Too many variables to be optimized: VAR coefficients α_{jk}^i , binary control variables γ_j^i and δ_{jk}^i .

(Note that, though $|\alpha_{jk}^i| \geq \eta_j^i \delta_{jk}^i$ and $\alpha_{jk}^i(1 - \delta_{jk}^i) = 0$ are also nonlinear, [E. Carrizosa, et al. 2017] provides linearized reformulation for them.)

Correlation-constrained SC-VAR (CCSC-VAR) model

Incorporate explicit spatial correlation information into the constraints!

$$\min_{\alpha, \delta} \sum_{i=1}^N \sum_{t=p}^T \left(y_{i,t+1} - \sum_{j=1}^N \sum_{k=1}^p \alpha_{jk}^i y_{j,t-k+1} \right)^2$$

subject to $\delta_{jk}^i \leq \lambda_j^i, \forall k \in \mathbf{K}, i, j \in \mathbf{I}$

$$\sum_{k=1}^p \delta_{jk}^i \geq \lambda_j^i, \forall i, j \in \mathbf{I}$$

$$\sum_{j=1}^N \sum_{k=1}^p \delta_{jk}^i \leq S_N^i, \forall i \in \mathbf{I}$$

$$|\alpha_{jk}^i| \leq M \cdot \delta_{jk}^i, \forall k \in \mathbf{K}, i, j \in \mathbf{I}$$

$$\delta_{jk}^i, \gamma_j^i \in \{0, 1\}, \forall k \in \mathbf{K}, i, j \in \mathbf{I}$$

where

$$\lambda_j^i = \begin{cases} 1, & \phi_j^i \geq \tau \\ 0, & \phi_j^i < \tau \end{cases}$$

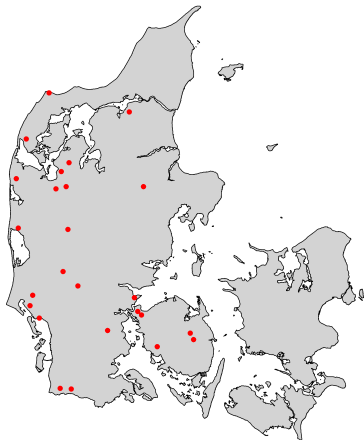
$$|\alpha_{jk}^i| \leq M \cdot \delta_{jk}^i \Leftrightarrow \begin{cases} -M \leq \alpha_{jk}^i \leq M, & \delta_{jk}^i = 1 \\ \alpha_{jk}^i = 0, & \delta_{jk}^i = 0 \end{cases}$$

Notations:

- ϕ_j^i is the Pearson correlation between wind farms i and j .
- M is a positive constant number (Generally $M < 2$).
- τ and S_N^i are used to control sparsity.

Improvements: *(simpler but better!)*

- Less parameters need to be tuned while the sparsity-control ability is preserved.
- More capable of characterizing the true inter-dependencies between wind farms.
- Less variables to be optimized.
- All constraints are linear.
- The model is decomposable.



- 25 wind farms randomly chosen over western Denmark
- 15-minute resolution
- 20.000 data points for each wind farm

Compared Models:

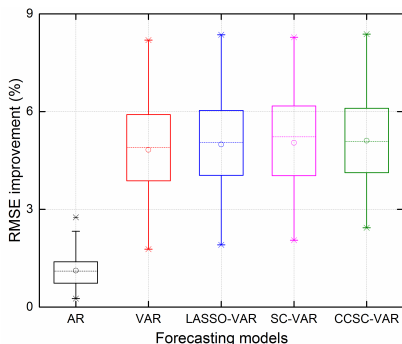
- **Local forecasting models**
 - Persistence method
 - Auto-Regressive model
- **Spatio-temporal models**
 - VAR model
 - LASSO-VAR model
 - SC-VAR model
 - CCSC-VAR model

Performance Metrics:

- Root Mean Square Error (RMSE)
- Mean Absolute Error (MAE)
- Sparsity for spatial models

Table: The average RMSE and MAE for all 25 wind farms for different forecasting models

Metrics	Persistence	AR	VAR	LASSO-VAR	SC-VAR	CCSC-VAR
Average RMSE	0.34843	0.34465	0.33156	0.33100	0.33080	0.33058
Average MAE	0.22158	0.22718	0.22631	0.22557	0.22490	0.22408
Model Sparsity	n/a	n/a	0	0.9248	0.8100	0.7504



RMSE improvement over Persistence method

From the Table and boxplot:

- All of the spatio-temporal models significantly outperform the local models.
- LASSO-VAR has highest sparsity but lowest accuracy among sparse models.
- CCSC-VAR model has lowest sparsity
- CCSC-VAR model has lowest average RMSE error for 25 wind farms
- The minimum, maximum and average improvements of CCSC-VAR are highest among these models.

- 8 Online sparse and adaptive learning for VAR models

Power output depends on previous outputs at the wind farm itself and other wind farms:

$$\mathbf{y}_n = \sum_{l=1}^L \mathbf{A}_l \mathbf{y}_{n-l} + \epsilon_n$$

Minimize

$$\sum_{t=1}^T \left\| \sum_{l=1}^L (\mathbf{A}_l \mathbf{y}_{n-l}) - \mathbf{y}_n \right\|_2^2$$

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Minimize

$$\sum_{t=1}^T \left\| \sum_{l=1}^L (\mathbf{A}_l \mathbf{y}_{n-l}) - \mathbf{y}_n \right\|_2^2 + \lambda \sum_{l=1}^L \|\mathbf{A}_l\|$$

- sparse coefficient matrices \mathbf{A}_l

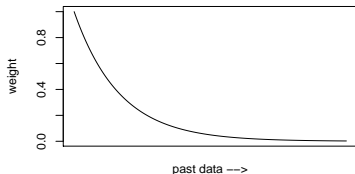
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$$\mathbf{y}_n = \sum_{l=1}^L \mathbf{A}_l \mathbf{y}_{n-l} + \epsilon_n$$

Minimize

$$\sum_{t=1}^T \nu^{N-n} \left\| \sum_{l=1}^L (\mathbf{A}_l \mathbf{y}_{n-l}) - \mathbf{y}_n \right\|_2^2 + \lambda \sum_{l=1}^L \|\mathbf{A}_l\|$$

- sparse coefficient matrices \mathbf{A}_l
- time-adaptive coefficients



Cyclic coordinate descent algorithm:

cyclically update coefficients until convergence:

$$A_l[i, j] \leftarrow \frac{\text{sign}(K_N)(|K_N| - \lambda)_+}{L_N}$$

$$K_N = \sum_{n=1}^N \nu^{N-n} y_{n-l}[j] (y_n[i] - \hat{y}_n[i] + A_l[i, j] y_{n-l}[j])$$

$$L_N = \sum_{n=1}^N \nu^{N-n} y_{n-l}[j]^2$$

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$$A_l[i, j] \leftarrow \frac{\text{sign}(K_N)(|K_N| - \lambda)_+}{L_N}$$

$$\begin{aligned} K_N &= \sum_{n=1}^N \nu^{N-n} y_{n-l}[j] (y_n[i] - \hat{y}_n[i] + A_l[i, j] y_{n-l}[j]) \\ &= \nu K_{N-1} + y_{N-l}[j] (y_N[i] - \hat{y}_N[i] + A_l[i, j] y_{N-l}[j]) \\ L_N &= \sum_{n=1}^N \nu^{N-n} y_{n-l}[j]^2 \\ &= \nu L_{N-1} + y_{N-l}[j]^2 \end{aligned}$$

→ data need not to be stored

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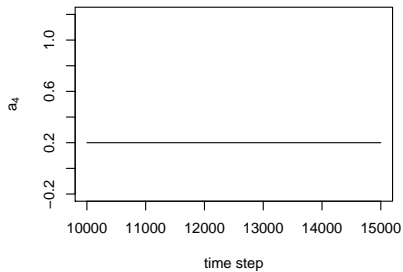
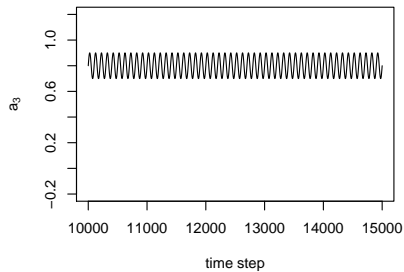
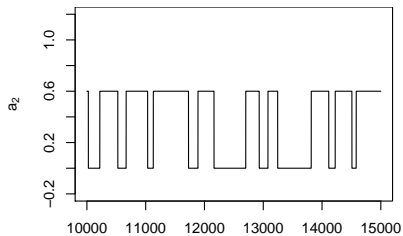
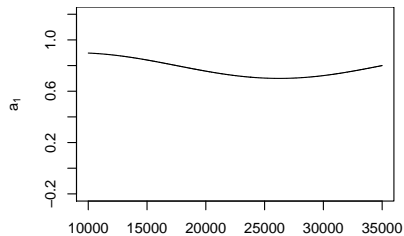
- data need not to be stored
 - initialize coordinate descent with previous estimates
- fast convergence

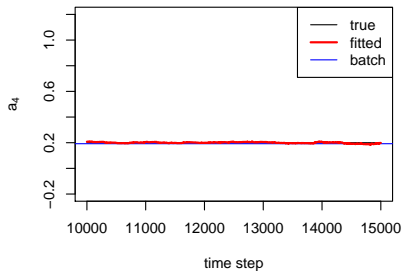
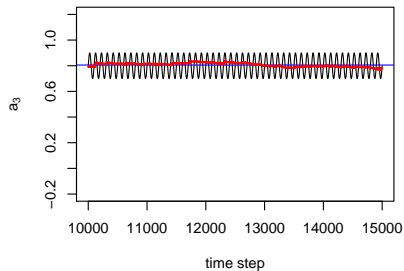
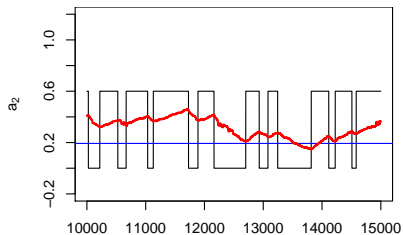
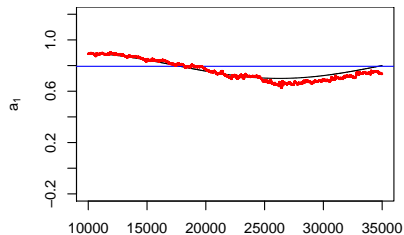
1st-order VAR time-series with coefficient matrix

$$\mathbf{A} = \begin{bmatrix} 0.9 & 0 & 0.1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & a_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_2 & 0.9 & 0 & 0 & 0 & 0.2 & 0 & 0 \\ 0.1 & 0 & 0 & 0 & a_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_4 & 0 & 0.9 & 0 & 0 & 0 & -0.1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.7 & 0 & 0 \\ 0 & 0 & -0.1 & 0 & 0 & 0 & 0 & 0 & 0.9 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.9 \end{bmatrix}$$

and a white multivariate Gaussian noise.

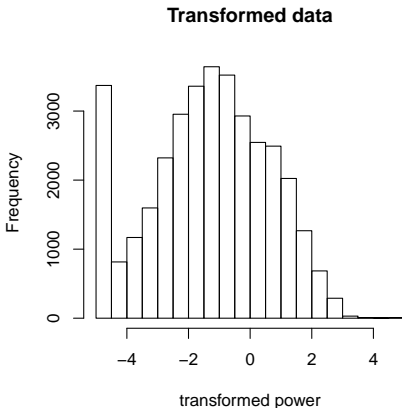
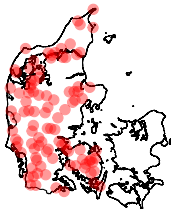
→ The interesting aspect is that a_1 , a_2 , a_3 , a_4 are time varying...



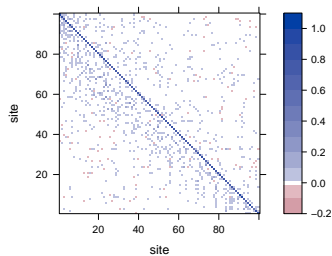


Sparsity: 49% (true: 83%)

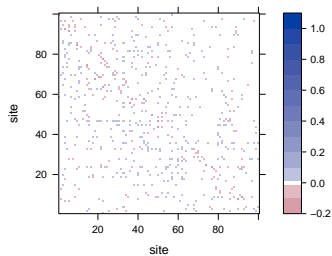
- 100 wind farms (out of 349), 15-min resolution
- logistic transformation
- 2011 (35.036 time steps)
- batch VAR estimation: first 20.000 data
- sorted from West to East



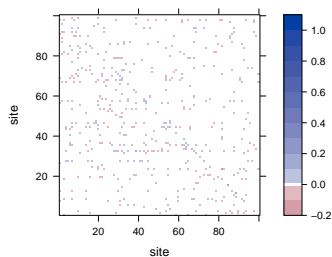
Lag-1 coefficient matrix



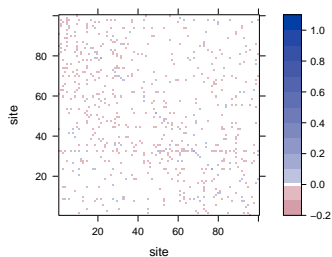
Lag-2 coefficient matrix

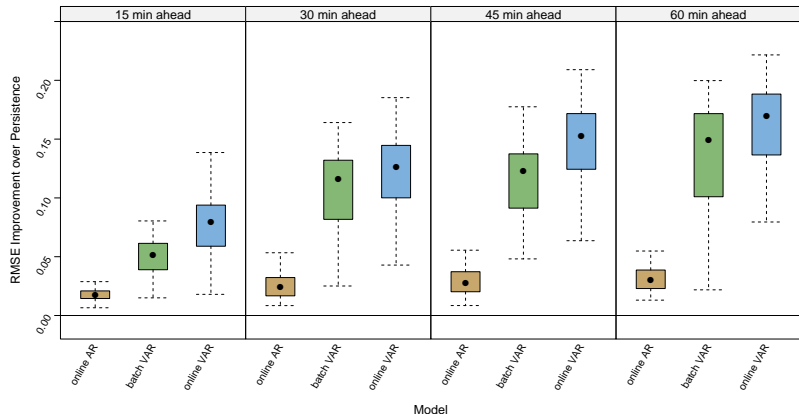


Lag-3 coefficient matrix



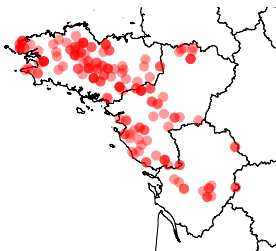
Lag-4 coefficient matrix



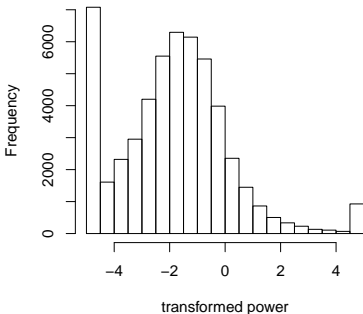


- the VAR model with batch learning outperformed AR models with online learning
- online sparse learning for the VAR model yields substantial extra gains

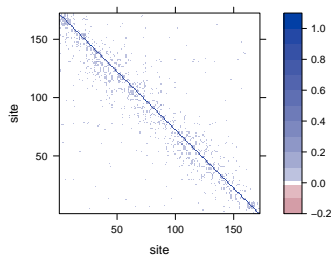
- 172 wind farms, 10-min resolution
- subset 2013 (52.561 time steps)
- logistic transformation
- batch VAR estimation: first 20.000 time steps
- sorted from West to East



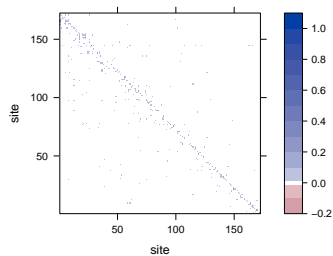
Transformed data



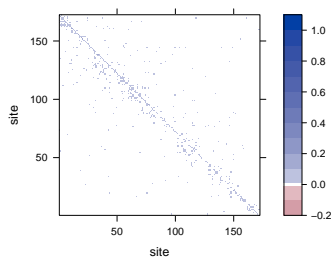
Lag-1 coefficient matrix

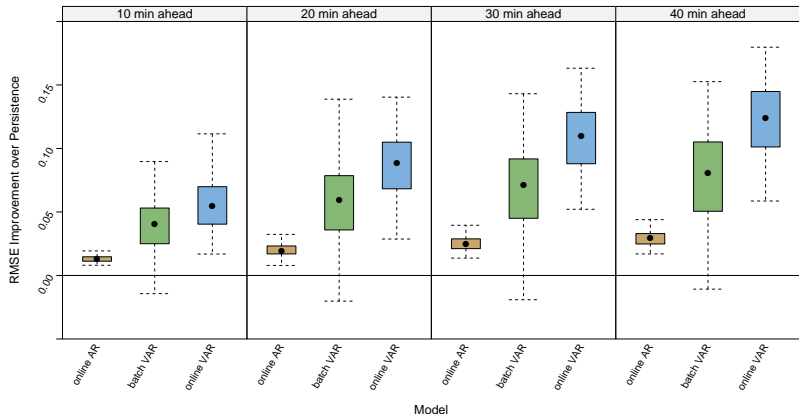


Lag-2 coefficient matrix

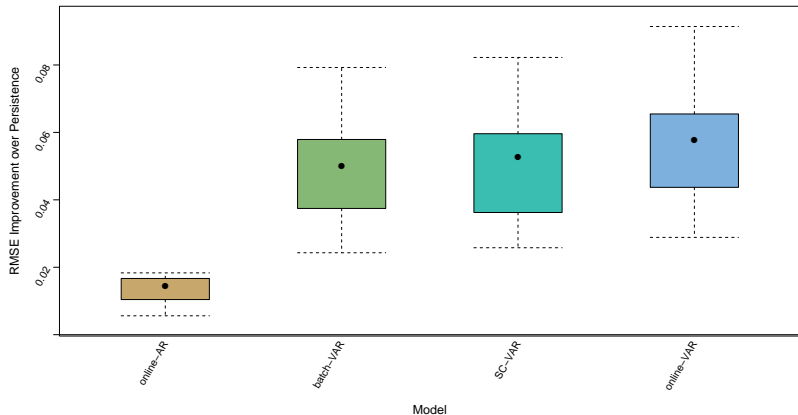


Lag-3 coefficient matrix





- the results obtained on the Danish data are confirmed with the French dataset...

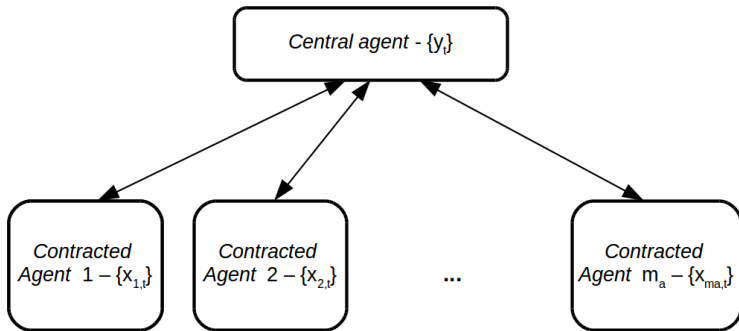


- the CCSC-VAR outperforms (slightly) the basic VAR with batch learning
- the online sparse VAR estimator does even better

4 Distributed learning

Data sharing... or not!

- To my knowledge, most players do not want to share their data - even though models and forecasts would highly benefit from that!
- one may design **distributed learning algorithms** that are **privacy-preserving**
- Example setup, with a *central* and *contracted* agents:



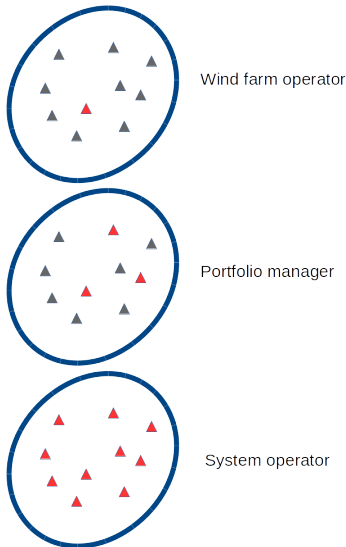
- Distributed learning, optimization, etc. is to **play a key role** in future energy analytics

- Wind power generation measurements $x_{j,t}$ are being collected at sites s_j , $j = 1, \dots, m$ (with t the time index)

- Out of the overall set of wind farms Ω ,
 - a **central agent** is interested in a subset of wind farms Ω_p (dim. m_p)
 - **contracted agents** relate to another subset of wind farms Ω_a (dim. m_a)

Write y_t the wind power production the central agent is interested in predicting

- 3 possible cases in practice:
 - a *wind farm operator* contracting neighbouring wind farms ($m_p = 1$)
 - a *portfolio manager* contracting other wind farms ($m_p > 1$)
 - a *system operator* interested in the aggregate production of all wind farms ($m_p = m$)



- Since restricting ourselves to the very short term, Auto-Regressive (AR) models with offsite information are sufficient
- Such a model reads as

$$y_t = \beta_0 + \sum_{s_j \in \Omega_p} \sum_{\tau=1}^l \beta_{j,\tau} x_{j,t-\tau} + \sum_{s_j \in \Omega_a} \sum_{\tau=1}^l \beta_{j,\tau} x_{j,t-\tau} + \varepsilon_t$$

where τ is a lag variable ($\tau = 1, \dots, l$)

- In a compact form:

$$y_t = \beta \mathbf{x}_t + \varepsilon_t$$

- As the number of coefficients may be large, we use a Lasso-type estimator, i.e.,

$$\hat{\beta} = \underset{\beta}{\operatorname{argmin}} \frac{1}{2} \|\mathbf{y} - \mathbf{A}\beta\|_2^2 + \lambda \|\beta\|_1$$

- After estimating β a forecast is given by

$$\hat{y}_{t+1|t} = \hat{\beta} \mathbf{x}_{t+1}$$

- The **Alternating Direction Method of Multipliers** (ADMM), is a widely used decomposition approach that allows to split a learning problem among features
- The Lasso estimation problem is first reformulated as

$$\begin{aligned} \min \quad & \frac{1}{2} \|\mathbf{y} - \mathbf{A}\boldsymbol{\beta}\|_2^2 + \lambda \|\mathbf{z}\|_1 \\ \text{s.t.} \quad & \boldsymbol{\beta} - \mathbf{z} = \mathbf{0} \end{aligned}$$

- It is then split among agents by setting

$$\begin{aligned} \boldsymbol{\beta} &= [\boldsymbol{\beta}_1, \boldsymbol{\beta}_2, \dots, \boldsymbol{\beta}_{m_a+m_p}] \\ \mathbf{A} &= [\mathbf{A}_1 \mathbf{A}_2 \dots \mathbf{A}_{m_a+m_p}] \end{aligned}$$

- The iterative solving approach is then defined such that, at iteration k ,

$$(\text{agent } j) \quad \boldsymbol{\beta}_j^k = \underset{\boldsymbol{\beta}_j}{\operatorname{argmin}} \left(\|\mathbf{A}_j \boldsymbol{\beta}_j - \mathbf{y}_j^{k-1}\|_2^2 + \frac{2\lambda}{\rho} \|\boldsymbol{\beta}_j\|_1 \right)$$

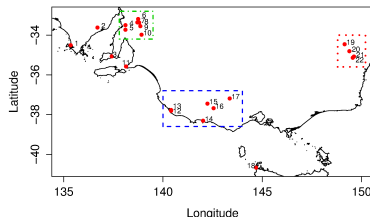
$$(\text{central agent}) \quad \bar{\mathbf{z}}^k = \frac{1}{(l+1)(m_a+m_p) + \rho} \left(\mathbf{y} + \overline{\mathbf{A}}\boldsymbol{\beta}^k \rho u^{k-1} \right)$$

$$\mathbf{u}^k = \rho u^{k-1} + \overline{\mathbf{A}}\boldsymbol{\beta}^k - \bar{\mathbf{z}}^k$$

(where $\mathbf{y}_j^{k-1} = \mathbf{A}_j \boldsymbol{\beta}_j^{k-1} - \overline{\mathbf{A}}\boldsymbol{\beta}^{k-1} + \bar{\mathbf{z}}^{k-1} - \mathbf{u}^{k-1}$, and $\overline{\mathbf{A}}\boldsymbol{\beta}^k = \sum_{j=1}^{m_a+m_p} \mathbf{A}_j \boldsymbol{\beta}_j$)

Australia

- Data from Australian Electricity Market Operator (AEMO)
- Data is public and shared by Uni. Strathclyde (Jethro Browell) and DTU
- 22 wind farms over a period of 2 years
- 5-minute resolution coarsened to 30 minutes



France

- Data from Enedis (formerly EDF Distribution)
- Data is confidential!
- 187 wind farms over a period of 3 years (only 85 used here)
- 10-minute resolution coarsened to 60 minutes

Only out-of-sample evaluation of genuine 1-step ahead forecasting!

Case 1: Wind farm operator

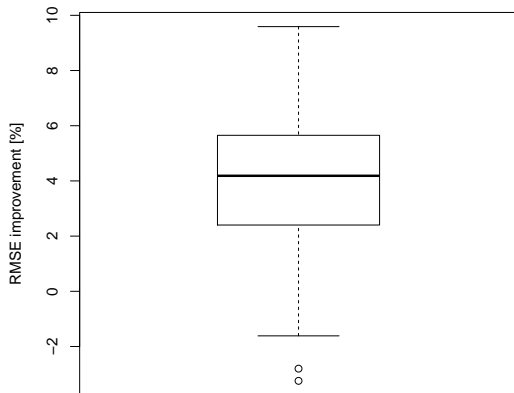
- Using Australian test case for a simple illustration at a single wind farm
- Comparison of persistence benchmark, local model (AR), and distributed learning model (ARX)

Table: Comparative results for distributed learning (ARX model), as well as persistence and AR benchmarks, at an Australian wind farm (wind farm no. 8) for 30-min ahead forecasting.

	Persistence	AR	ARX (dist. learning)
RMSE [% nom. capacity]	3.60	3.57	3.52
Improvement [%]	-	0.8	2.2

- The improvement is modest, but significant
- This is while the central agent (wind farm 8) never had access to data of contracted wind farms
- Thanks to L_1 -penalization, the number of contracted wind farm is very limited

- Extensive analysis based on the French dataset
- Improvement of distributed learning over local model only, in terms of RMSE



- Improvement is nearly always there
- It ranges from modest to substantial
- This obviously depends on the wind farm location

- Using French test case
- We randomly pick 8 wind farms to build a portfolio
- Comparison of persistence benchmark, local model (AR), and distributed learning model (ARX)

Table: Comparative results for distributed learning (ARX model), as well as persistence and AR benchmarks, for a portfolio of 8 wind farms of the French dataset (randomly chosen) for 1-hour ahead forecasting.

	Persistence	AR	ARX (dist. learning)
RMSE [% nom. capacity]	3.99	3.67	3.38
Improvement [%]	-	8.2	15.3

- The improvement is substantial
- Again, thanks to L_1 -penalization, the number of contracted wind farm is very limited
- Simulation studies may allow to look at how improvement relates to portfolio size, wind farm distribution, etc.

Case 3: System operator

- Using French test case
- The system operator aims to predict the aggregate of all wind farms, though never accessing the wind farm data(!)
- Comparison of persistence benchmark, local model (AR), and distributed learning model (ARX)

Table: Comparative results for distributed learning (ARX model), as well as persistence and AR benchmarks, for the aggregate of all 85 French wind farms for 1-hour ahead forecasting.

	Persistence	AR	ARX (dist. learning)
RMSE [% nom. capacity]	2.88	2.10	2.05
Improvement [%]	-	27.1	28.8

- The improvement is modest, since an AR model obviously does very well for aggregated wind power production
- Though, the practical interest is huge, since data does not need to be exchanged
- More complex models (e.g., regime-switching) may yield higher improvements

- **High-dimensional** and **distributed learning** have a **bright future in energy analytics**, since
 - high quantity of distributed data is being collected
 - data-driven and expert input to reveal and maintain sparsity
 - most actors do not want to share their data (unless forced to do so)
- Some interesting **future developments**:
 - online distributed learning (i.e., merger of ideas presented), to lighten computational costs and exchange/communication needs
 - broaden the applicability to a wide class of models, e.g., with regime switching and regression on input weather forecasts
 - design distributed computation and data sharing markets!

Thanks for your attention!

