# Strategic bidding in electricity markets

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# Strategic bidding in electricity markets:

# Background

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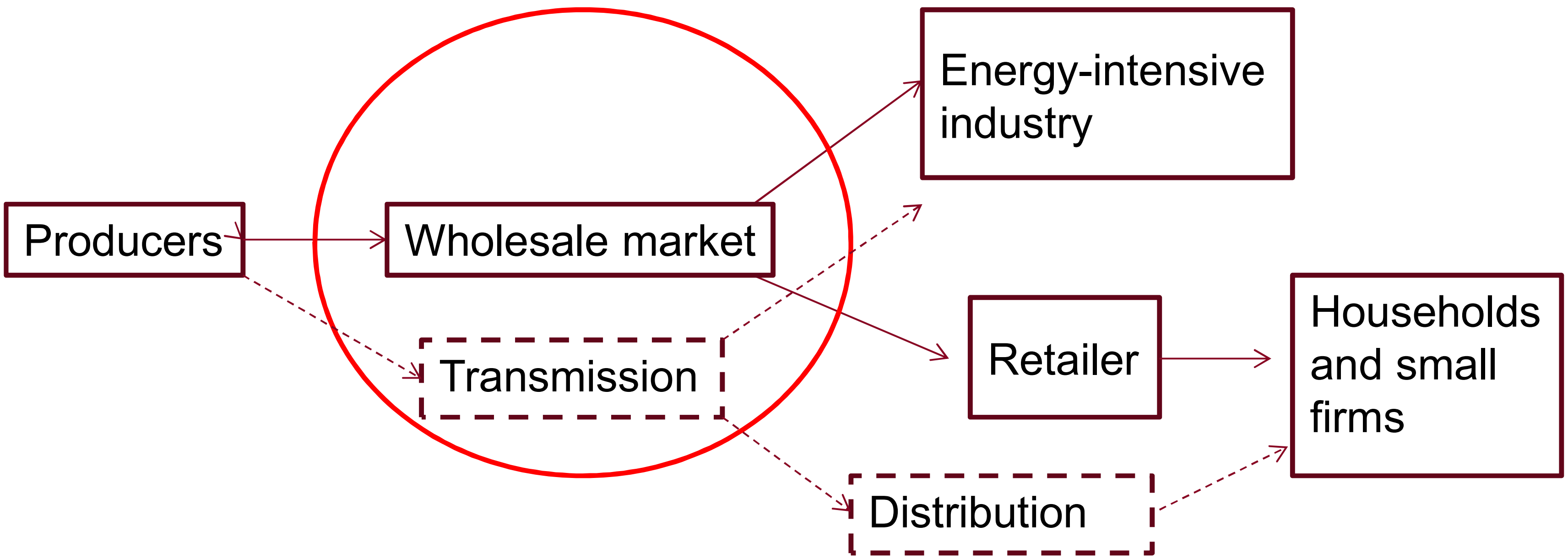
# Electricity market

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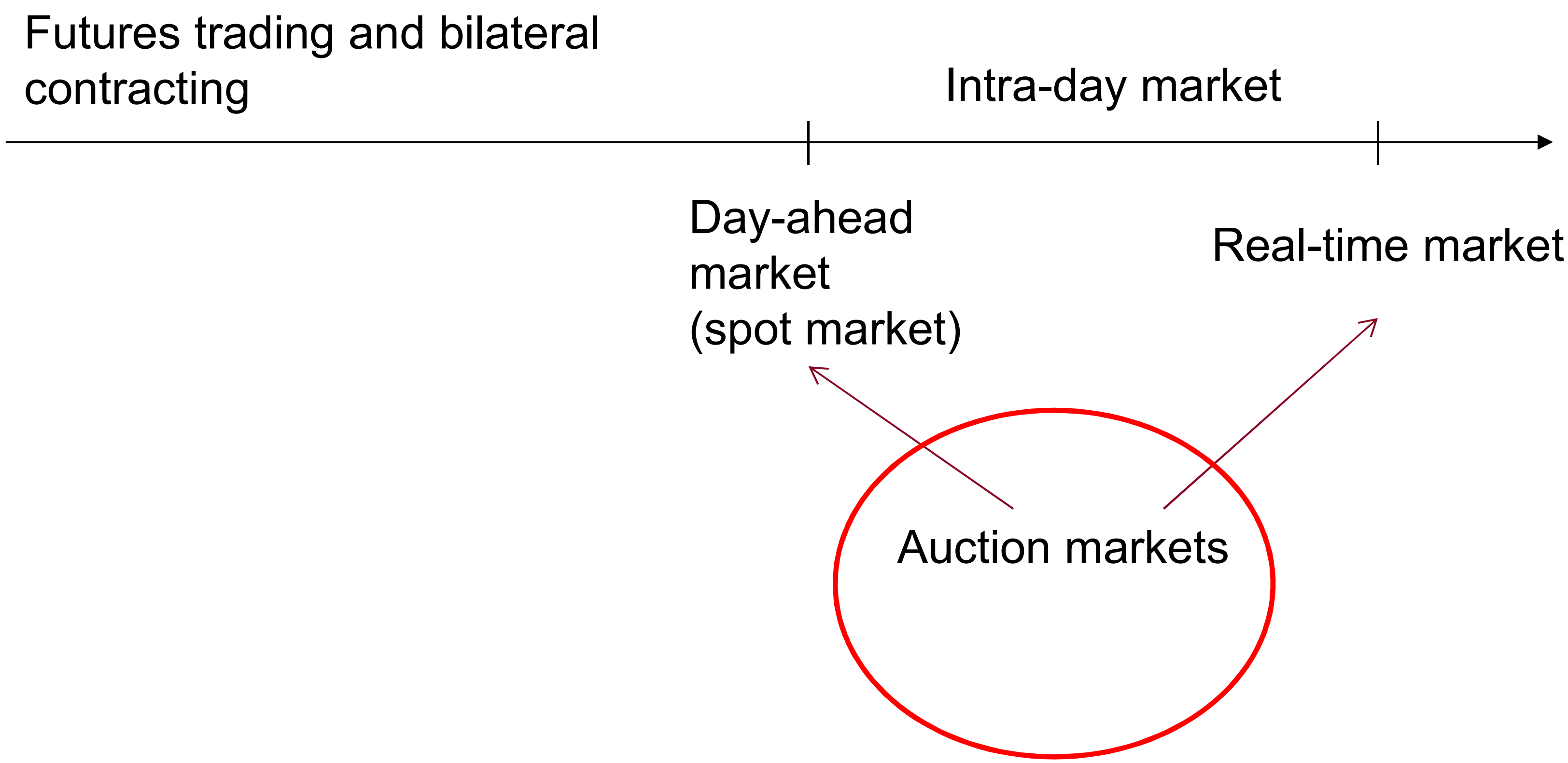
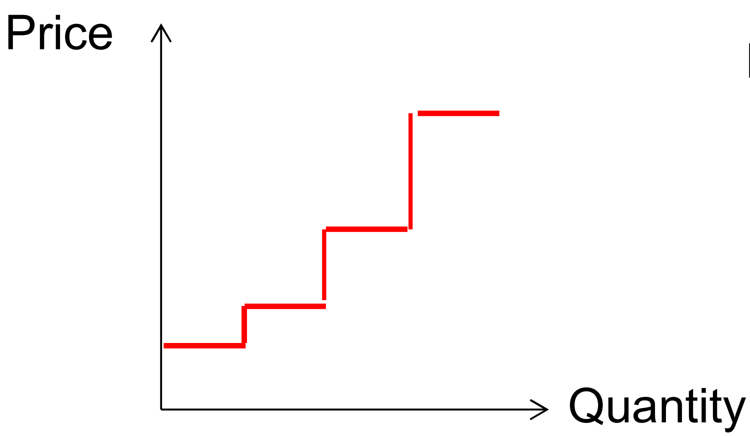
# Wholesale electricity market

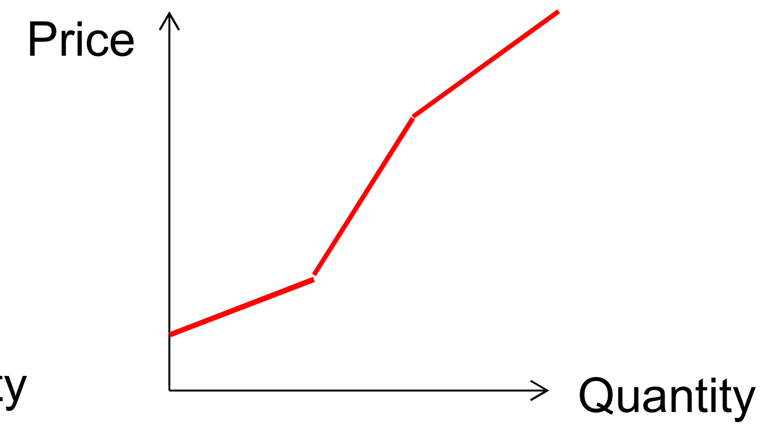



See Stoft (2002) for further details.

# Supply function offers

Producers use supply functions to inform market operator of their marginal costs.

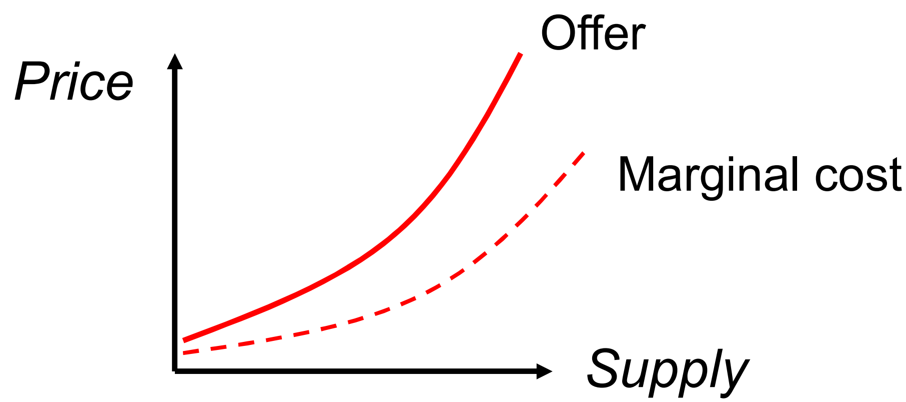



Typically offers are stepped or piece-wise linear

# Producers can bid strategically

Producers have incentives to overstate their costs in order to increase their profit.




How large are the mark-ups and how do they depend on competition, contracts, market design, network congestion etc.

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# Strategic bidding in electricity markets:

# Uniform-price auctions and the Supply Function Equilibrium (SFE)

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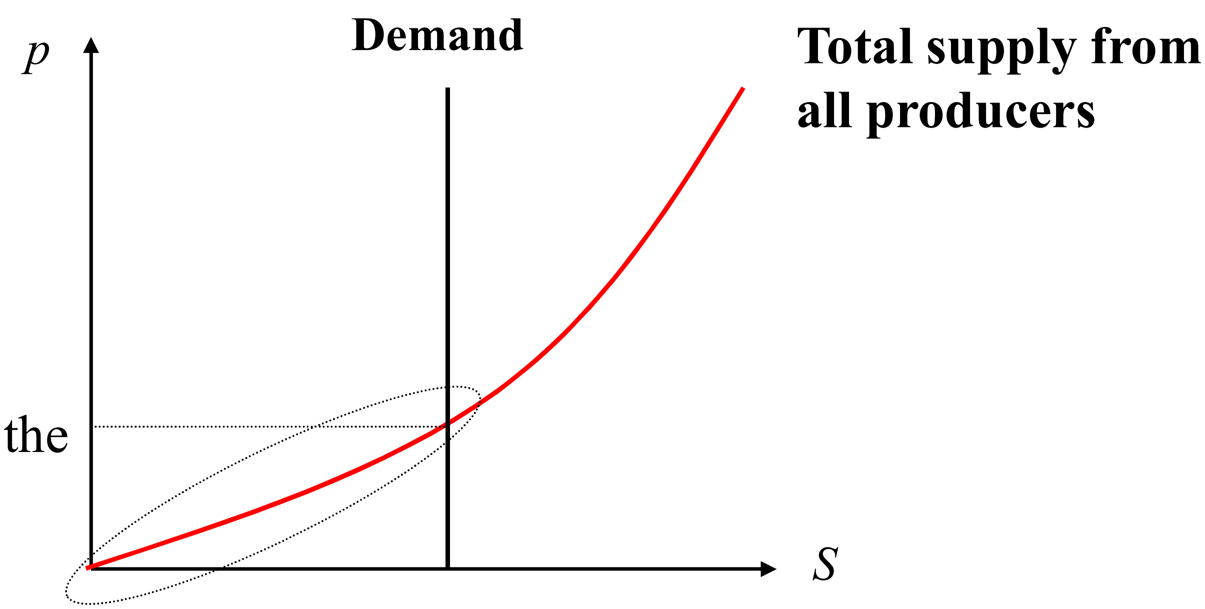
# Uniform pricing

Most wholesale electricity markets use uniform-pricing.




**Uniform-pric**e: All accepted bids are paid the price of the marginal offer.

# The supply function equilibrium (SFE)

**Behavioural assumption:** Each producer chooses its supply curve to maximize its expected profit.

**Game-theoretic model.** Nash equilibrium: every producer maximizes its expected profit given competitors' supply curves and properties of the uncertain demand. Equilibrium is called Supply Function Equilibrium (SFE).

Introduced by Klemperer & Meyer (1989). First application to electricity market by Green & Newbery (1992).

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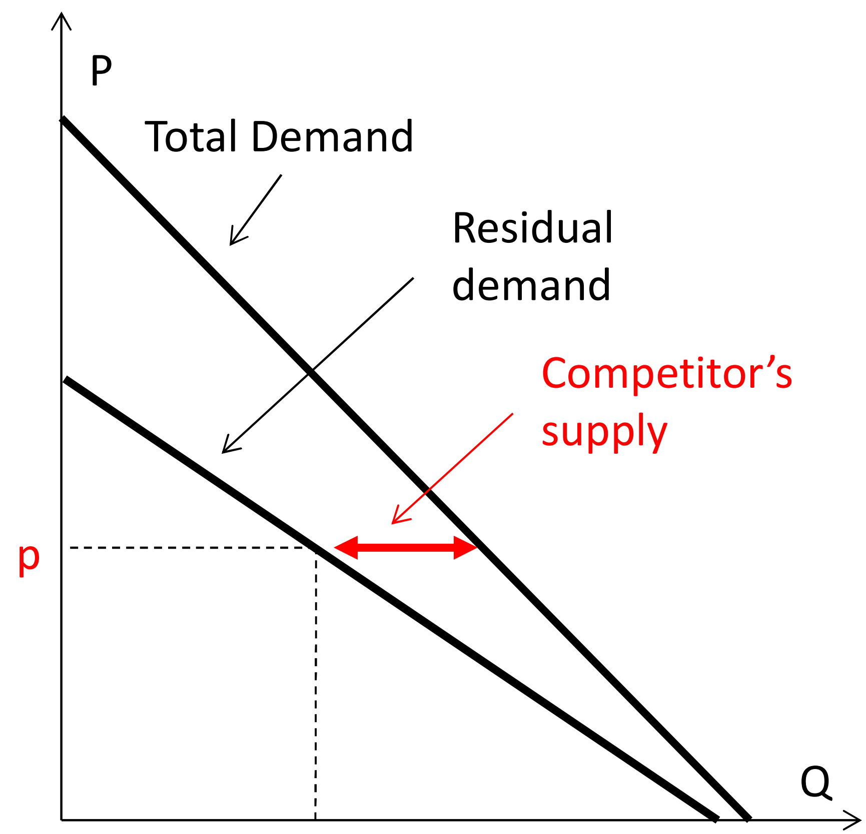
# Standard simplifying assumptions for SFE

- Production costs are well-known (≈ common knowledge)
- Few producers in the market => Market power
- Many consumers/retailers in the market => ≈Price takers
- Demand has additive demand shock *ε.*

# Residual demand

The residual demand curve is the individual firm's demand curve, i.e. its part of market demand that is not supplied by other firms in the market.

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# Optimal output

Residual demand of producer *i*:

$$R_i(p) = D(p) - \sum_{j \neq i} S_j(p)$$

Profit:

$$\Pi_i(p) = pR_i(p) - C_i(R_i(p))$$

Optimal profit:

$$\frac{\partial \Pi_i(p)}{\partial p} = \underbrace{R_i(p)}_{Price\ effect} + \underbrace{\left(p - C_i{}'(R_i(p))\right)\frac{\partial R_i(p)}{\partial p}}_{Quantity\ effect} = 0$$

At the clearing price, we have $S_i(p)$=$R_i(p)$ ⇒ First-order condition:

$$S_i(p) = \left(p - C_i{}'(S_i(p))\right)\left(\sum_{j \neq i} S_j{}'(p) - D'(p)\right)$$

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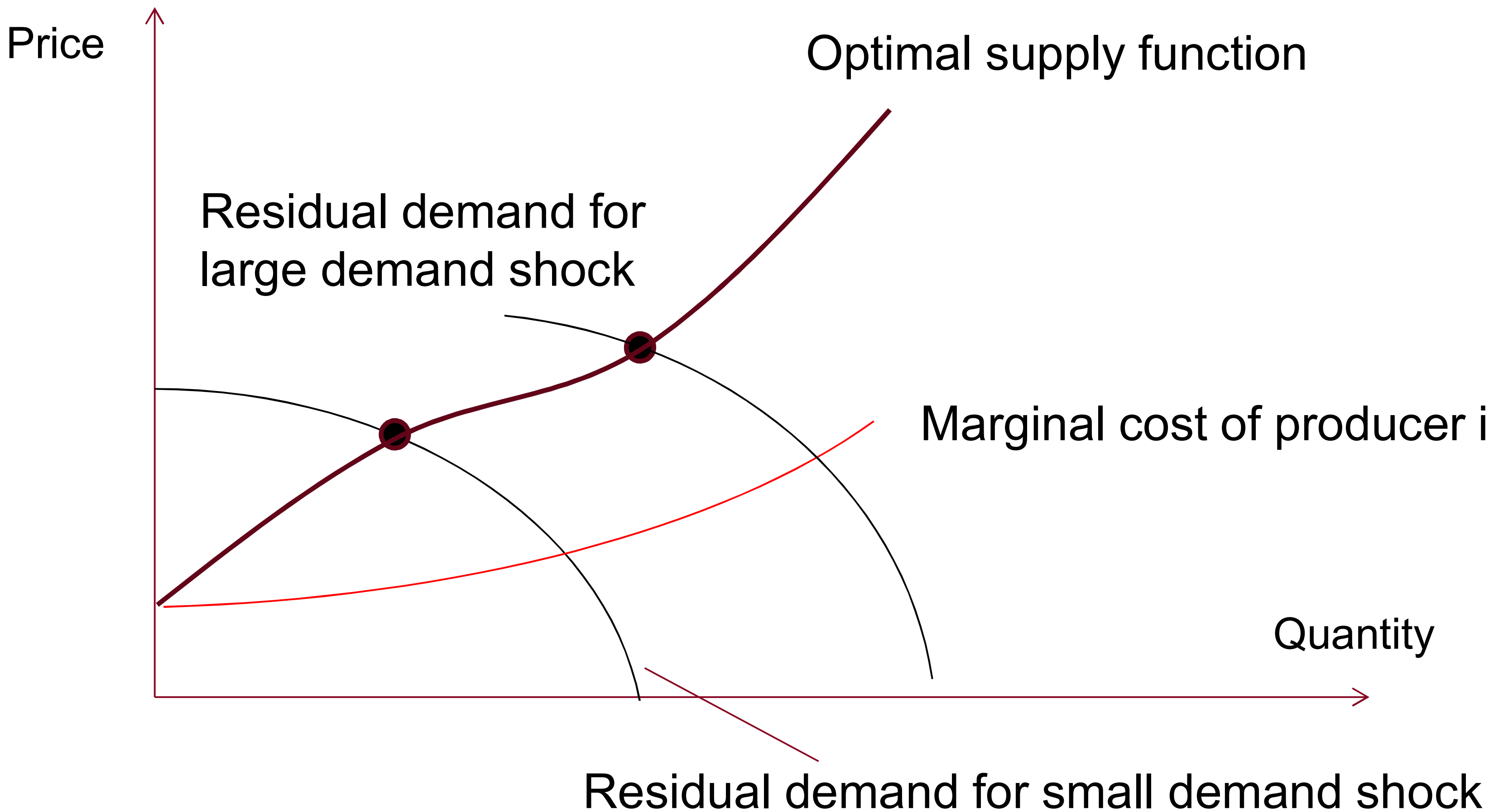
# Optimal supply function




Output is ex-post optimal (a producer would not change its mind after the shock is observed).

# Supply function equilibrium

SFE is determined from system of first-order conditions (one for each firm).

$$S_i(p) = \left(p - C_i{}'(S_i(p))\right)\left(D'(p) - \sum_{j \neq i} S_j{}'(p)\right)$$

If demand is downward sloping and marginal costs are up upward-sloping, then a set of upward sloping solutions to the system of first-order condition is an SFE; expected profits are globally maximized for each firm (Holmberg and Willems, 2015).

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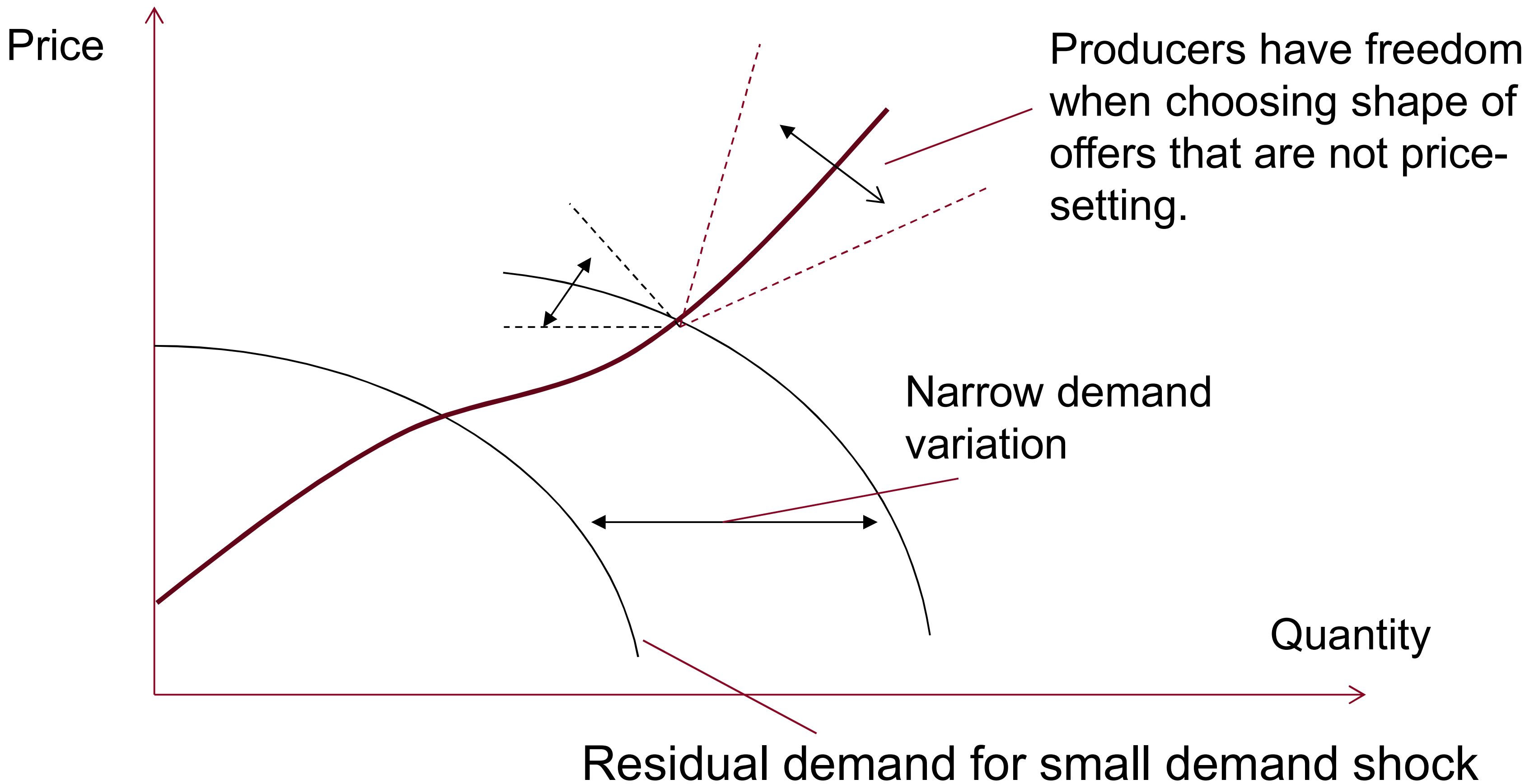
# Narrow demand range => Multiple SFE

Narrow support of demand shocks => Multiple SFE (Klemperer and Meyer, 1989; Green and Newbery, 1992; Genc and Reynolds, 2011)

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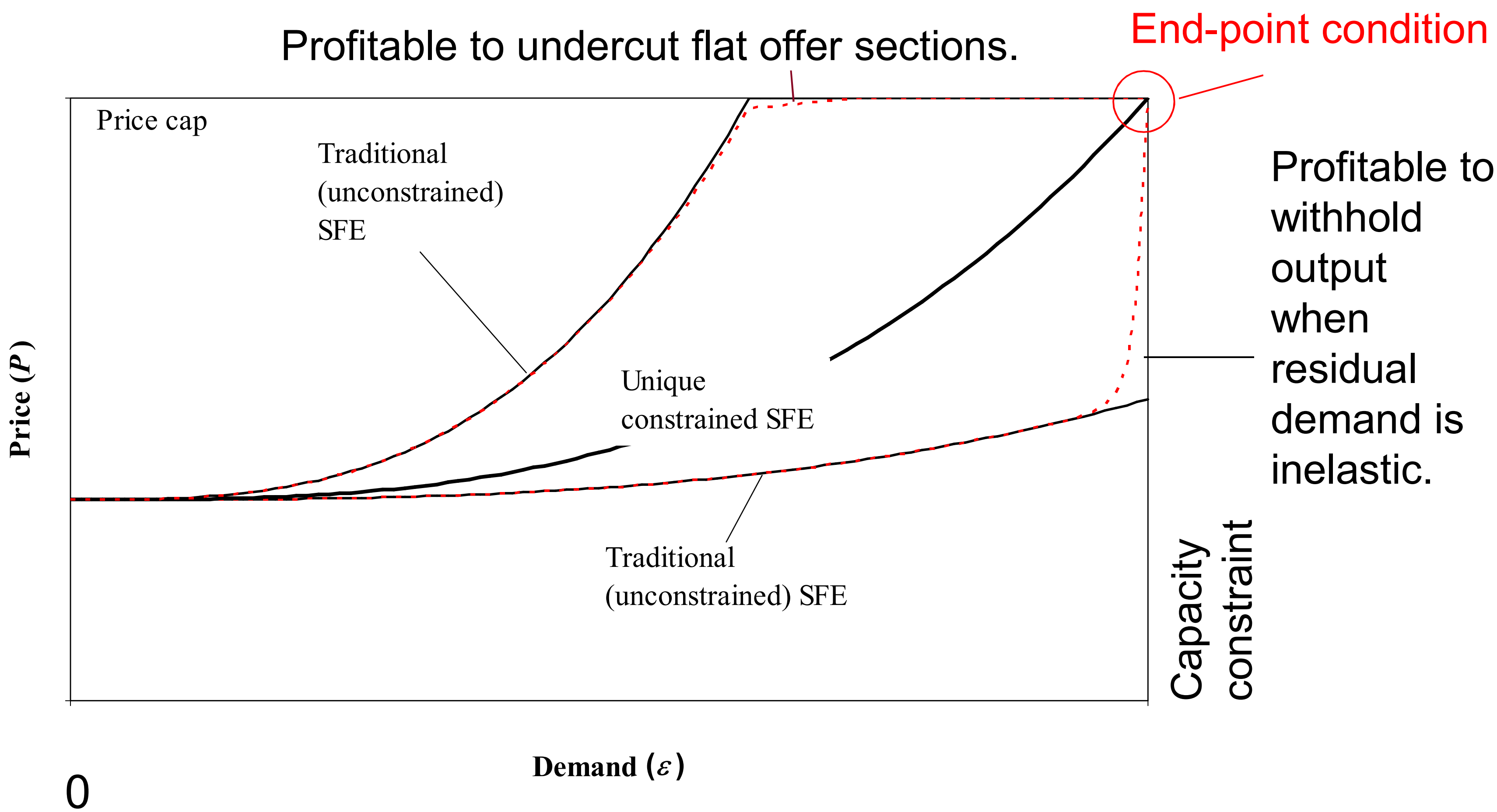
# Wide demand range => Unique SFE

Holmberg (2008) proves uniqueness of SFE for symmetric market with inelastic demand when the support of demand shocks is sufficiently wide.




Anderson (2013) proves uniqueness and existence for asymmetric duopoly market with elastic demand.

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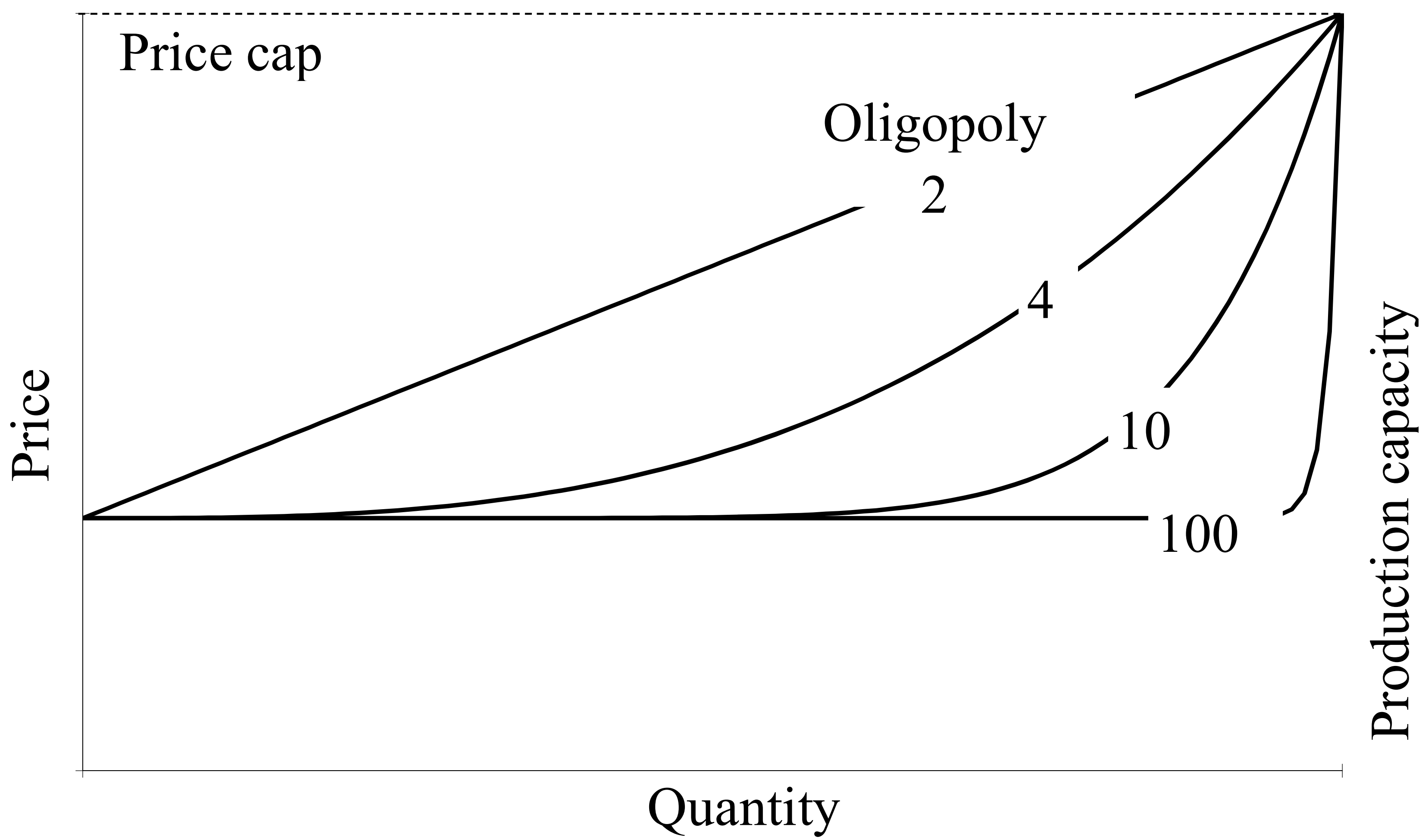
# Example: Symmetric firms with constant MC and inelastic demand




Holmberg (2008)

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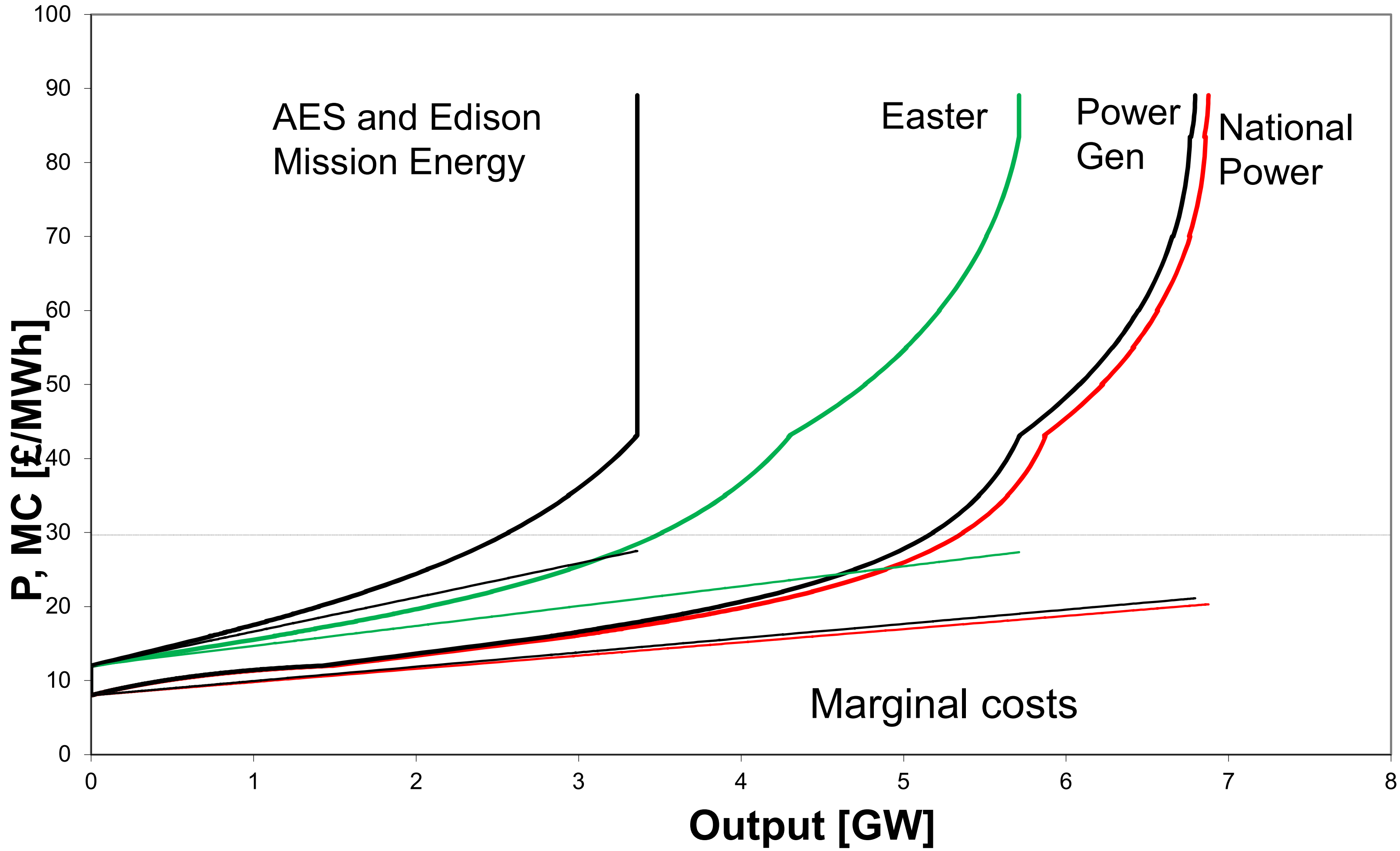
# Numerical computation of asymmetric SFE

Example from UK for 1999 (Anderson & Hu; 2008; Holmberg, 2009):




Further developed by Ruddell (2017)

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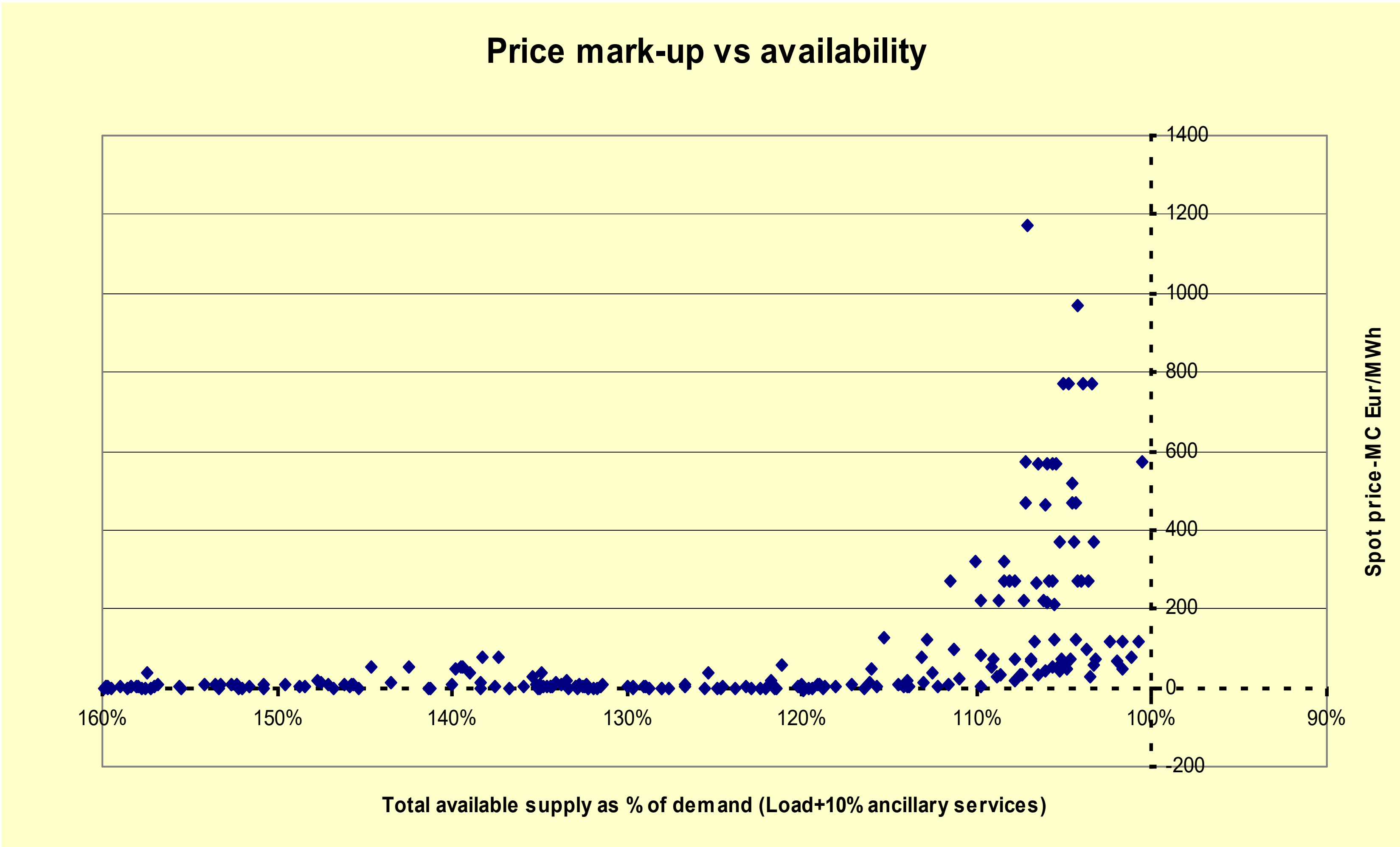
# Strategic bidding in practice

Market data => Producers in Australia (Wolak, 2003) and large producers in Texas (Hortacsu and Puller, 2008; Sioshansi and Oren, 2007) bid roughly as predicted by theory. Example from Europe below:



# Advantages with uniform-pricing

- Equilibrium bids are fairly robust to uncertainties; they are not sensitive to shocks in the auctioneer’s demand/supply
- Easy for small firms; it is optimal for them to simply bid their marginal cost.
- Gives a well-defined spot price that can be used to settle financial contracts and payments of non-competitive bidders.

# Cournot NE

In a Cournot model, offers are restricted to be independent of the price, i.e. $S_i{}'(p)$=0. Thus the first-order condition simplifies as follows:
$S_i = \left(p - C_i{}'(S_i)\right)D'(p)$.

Demand is normally assumed to be certain in Cournot models. The equilibrium output of each firm can be determined from a system of first-order conditions (one condition per firm).

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# Strategic bidding in electricity markets:

# Contracts

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# Optimal supply function with contracts

In electricity markets, producers typically hedge 80-90% of their planned output.

Let $F_i$ be the volume of producer *i,* for which the price has been fixed at $\bar{p}$.

Profit of producer *i*:

$$\Pi_i(p) = \bar{p}F_i + p(R_i(p) - F_i) - C_i(R_i(p))$$

Differentiation with respect to *p* =>

$$\frac{\partial \Pi_i(p)}{\partial p} = \underbrace{R_i(p) - F_i}_{Price\ effect} + \underbrace{\left(p - C_i{}'(R_i(p))\right)\frac{\partial R_i(p)}{\partial p}}_{Quantity\ effect} = 0$$

$S_i(p) = R_i(p)$=> First-order condition:

$$S_i(p) - F_i = \left(p - C_i{}'(S_i(p))\right)\left(D'(p) - \sum_{j \neq i} S_j{}'(p)\right)$$

See further details in Newbery (1998).

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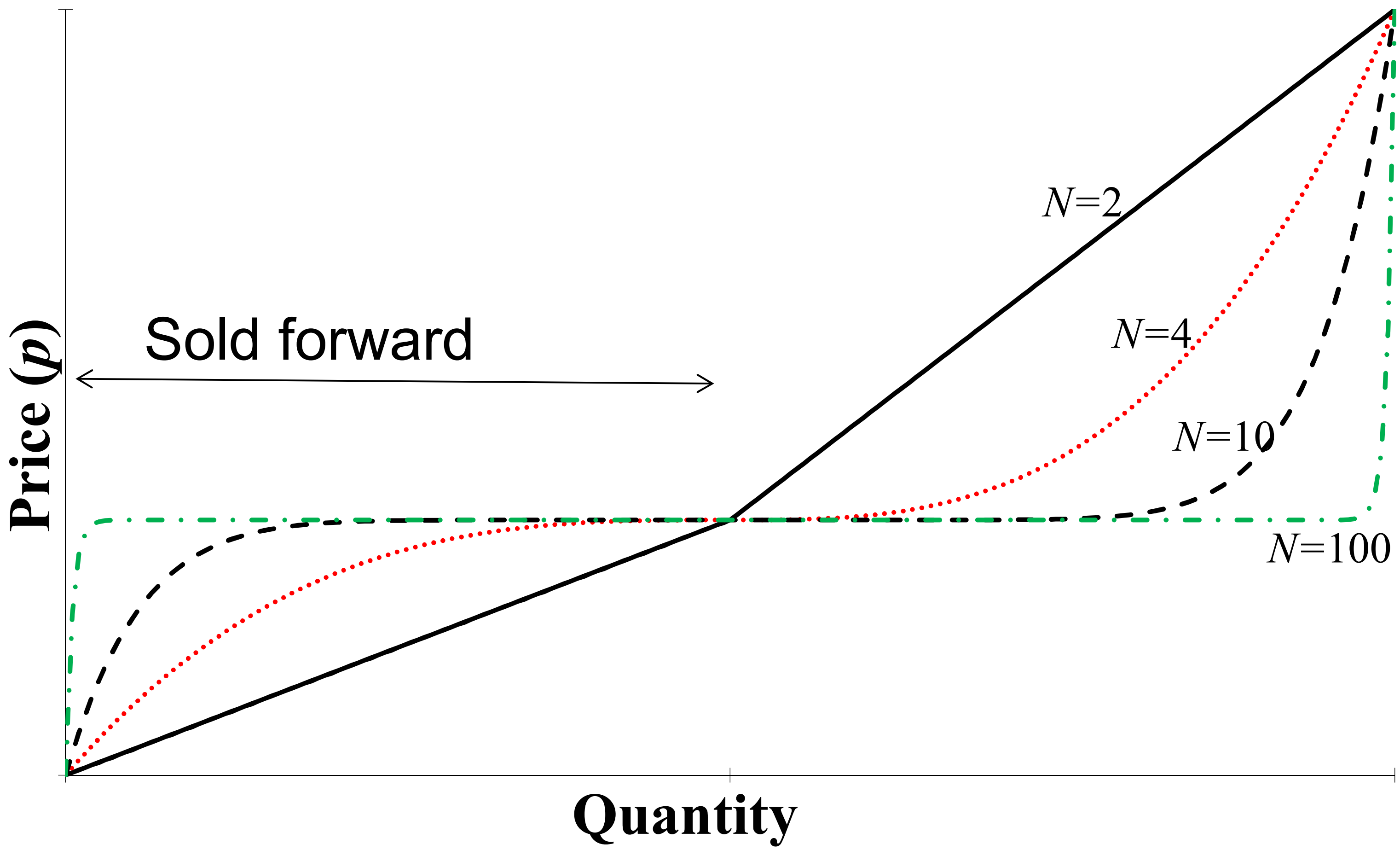
# Example with contracts




Forward sales make markets more competitive (Newbery, 1998).

# Strategic contracting

Contracting is useful to edge the profit, which is useful for risk-averse producers. Are there are also strategic reasons for selling forward contracts?

Hedging a large volume => a producer becomes less interested in increasing the price => a credible/rational commitment to increased output in the spot market. This could influence bidding of competitors if contracting is observed by competitors.

Allaz and Vila (1992) show that producers have strategic reasons to sell forward contracts in a Cournot model. A commitment to increased output => Reduced output of competitors in a Cournot model. => The introduction of forward tranding improves market performance.

Holmberg and Willems (2013) show that strategic contracting would worsen competition if producers can trade options contracts. The reason is that producers would find it profitable to commit to downward sloping supply functions.=> The introduction of options trading worsen marker performance.

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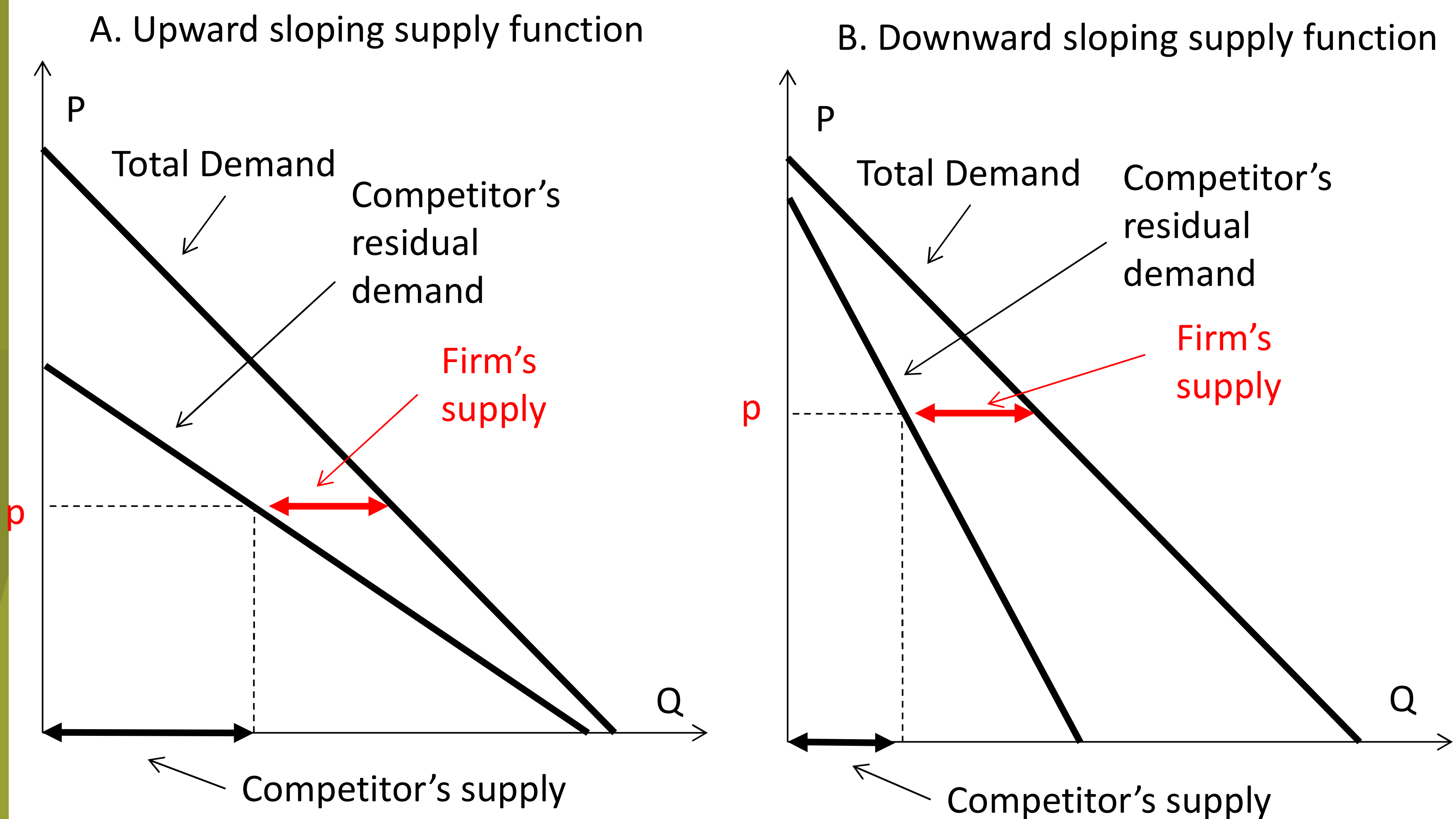
# Why do firms commit to a negative slope?




**Firm sells same amount at higher price**

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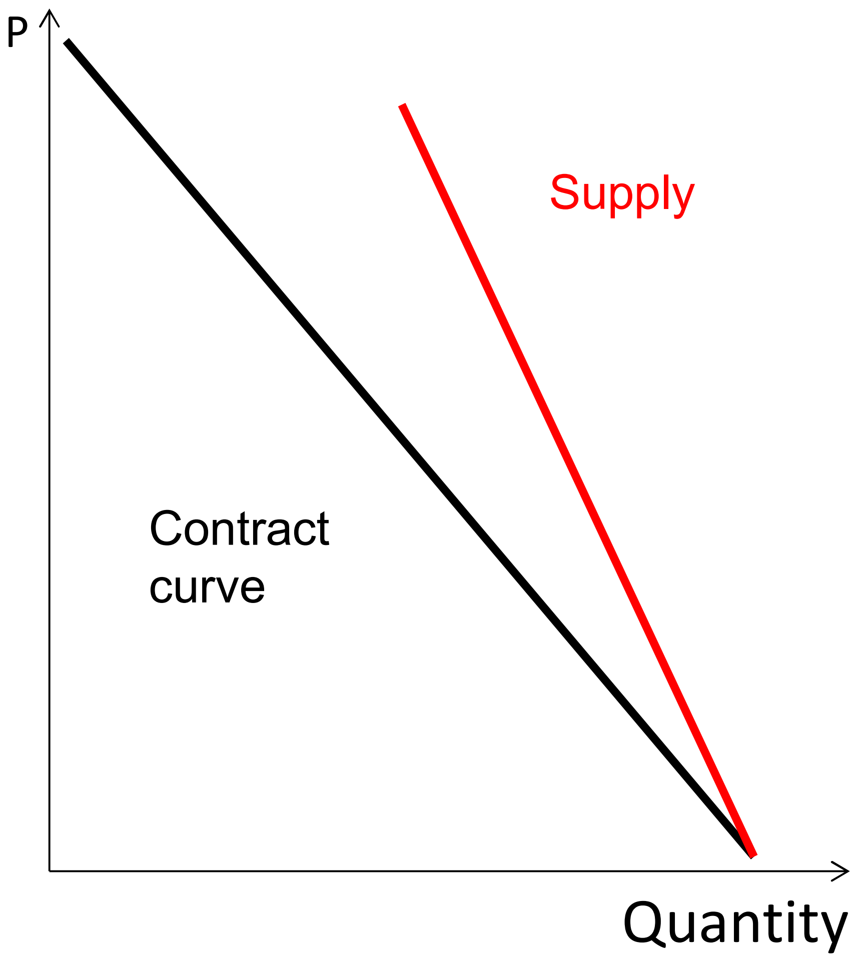
# How do firms commit to a downward sloping supply?




Make contract position a function of the price

- Large for low prices (aggressive commitment)
- Small for high price (soft commitment)

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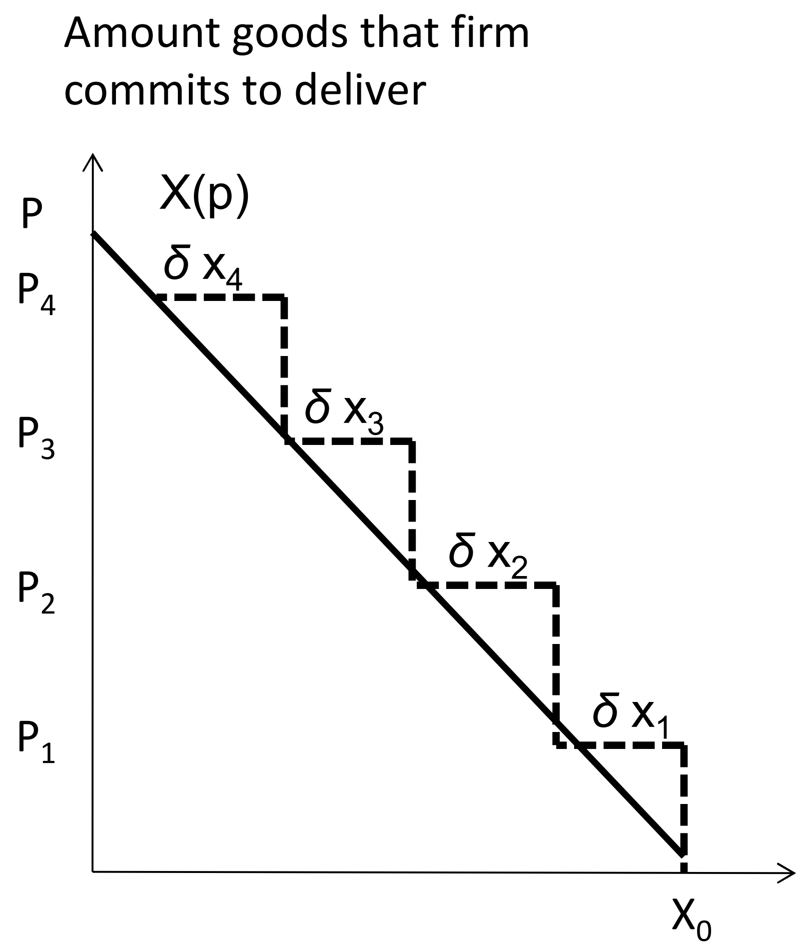
# How do firms commit to a downward sloping contracting curve?

**Sell $X_0$** forward contracts

**Buy $\delta X_1$** call options with strike price $P_1$

**Buy $\delta X_2$** call options with strike price $P_2$

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# Strategic bidding in electricity markets:

# The market distribution function

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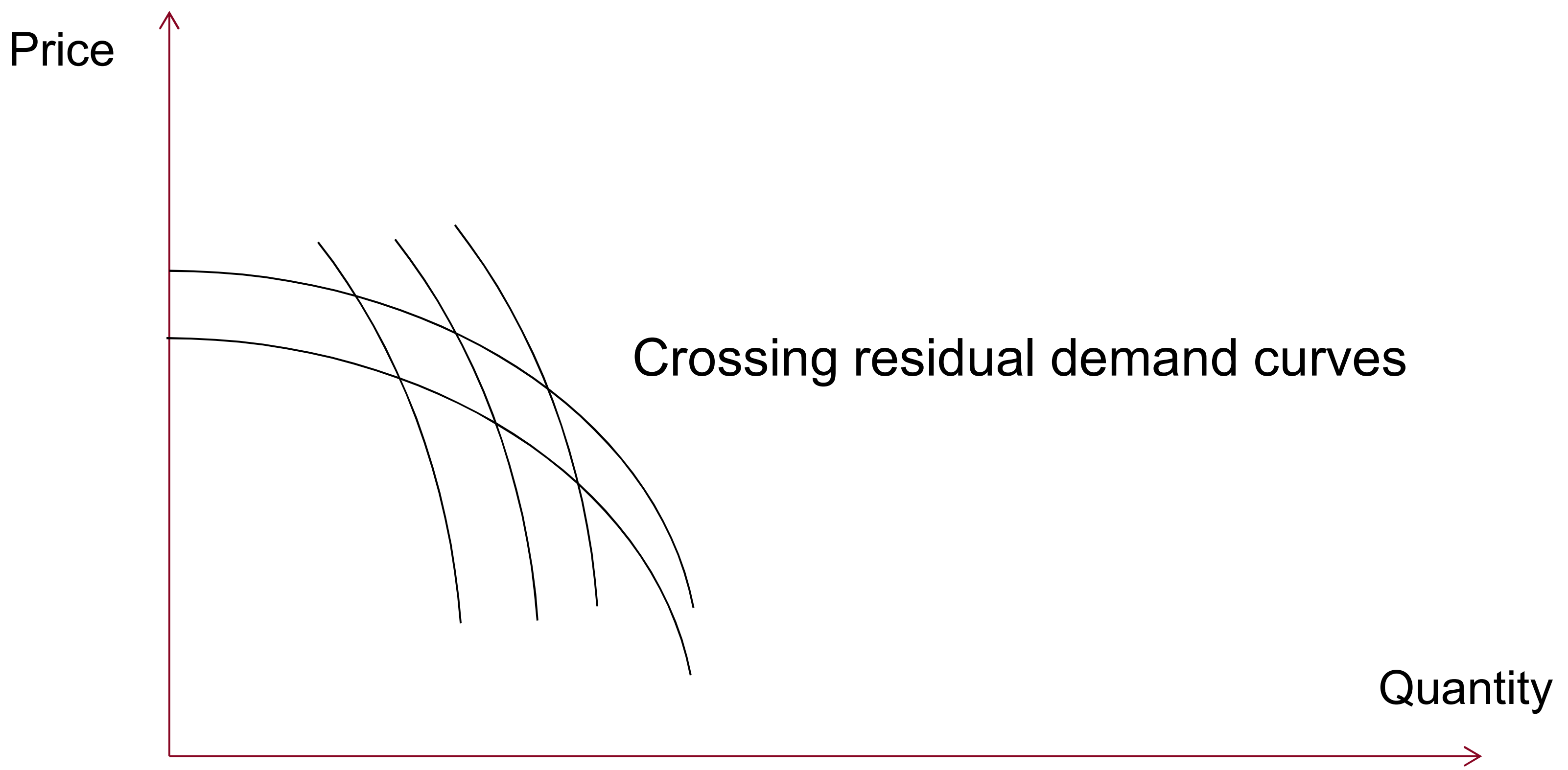
# Complicated/crossing residual demand curves

Non-crossing residual demand curves are straightforward as a producer can independently optimize its supply/price for each demand shock $\varepsilon$.Crossing residual demand curves are more complicated. Equilibria will be ex-ante optimal.

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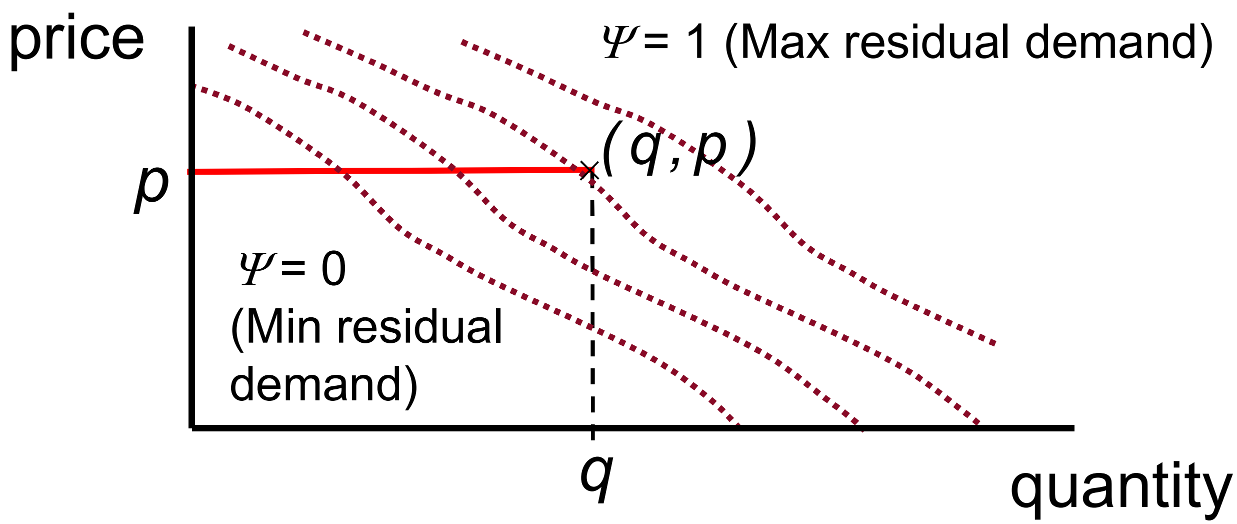
# The market distribution function

We let $\psi(q,p)$ be the market distribution function (Anderson and Philpott, 2002; Wilson, 1979). It is the probability that an offer *(q,p)* is rejected, i.e. the residual demand curve passes below *(q,p)*.

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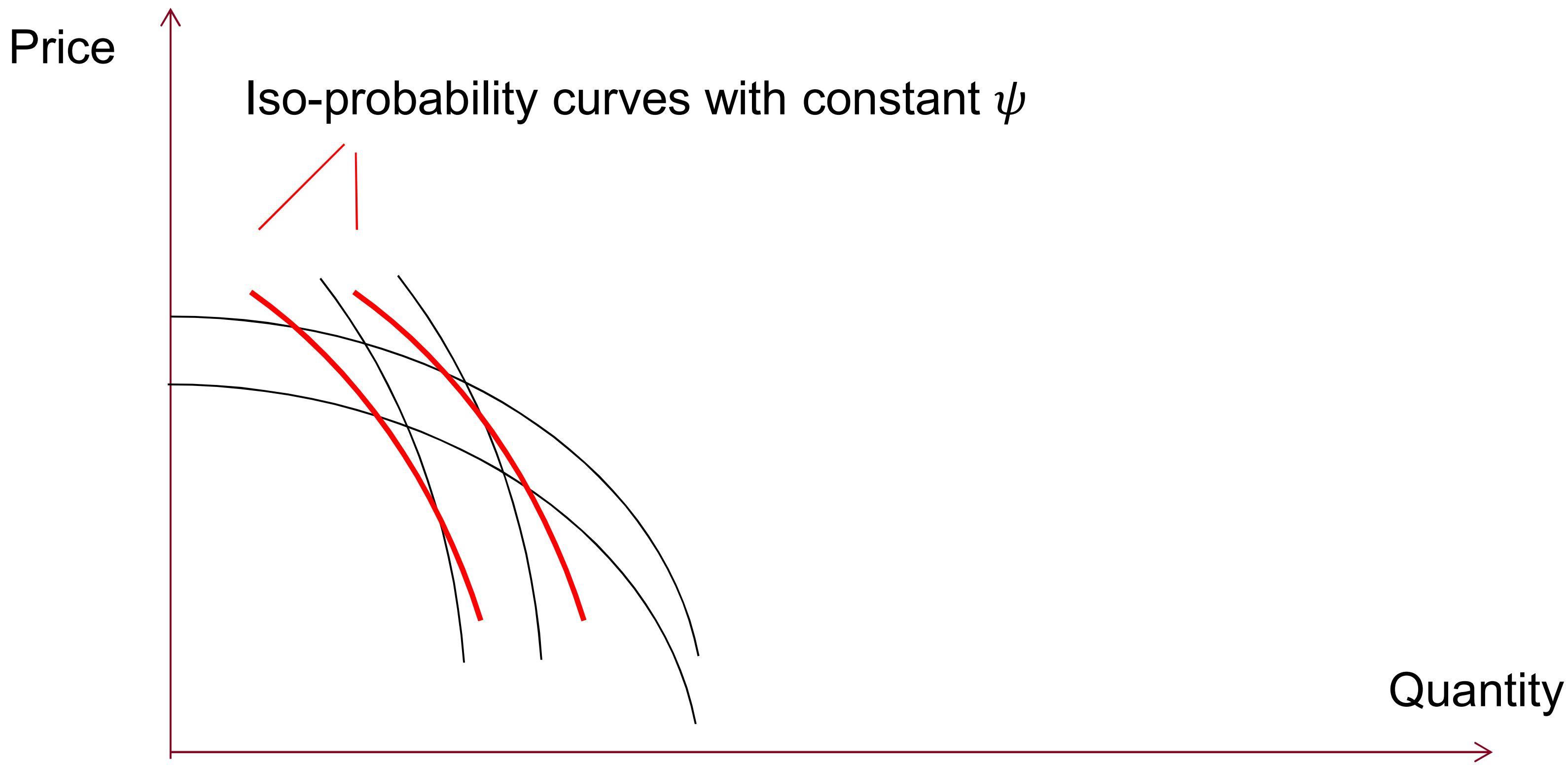
# Market distribution function for crossing residual demand




Anderson and Philpott (2002) show that:

$$\pi = \int_S \left(pq - C_i(q)\right) d\psi_i(p,q)$$

$\Rightarrow$ The expected profit does not depend on how $\psi_i(p,q)$ was generated => Choose non-crossing residual demand.

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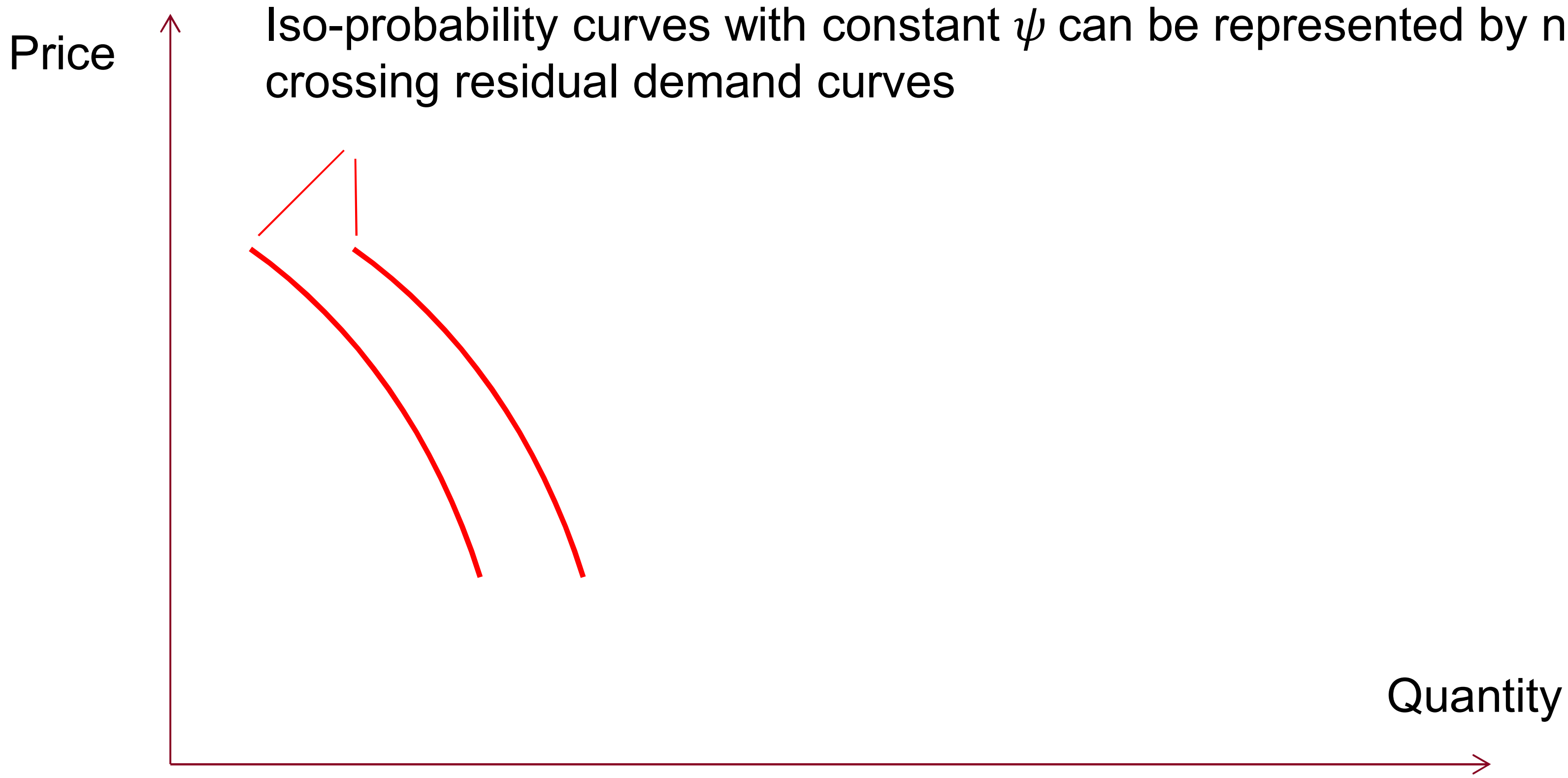
# Equivalent non-crossing residual demand




$$\frac{\partial R_i}{\partial p} = -\frac{\partial \psi / \partial p}{\partial \psi / \partial q}$$

FOC in uniform-price auction:

$$S_i(p) = -\left(p - C_i{}'(S_i(p))\right) \frac{\partial \psi / \partial p}{\partial \psi / \partial q}$$

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# Strategic bidding in electricity markets:

# Discriminatory (pay-as-bid) pricing

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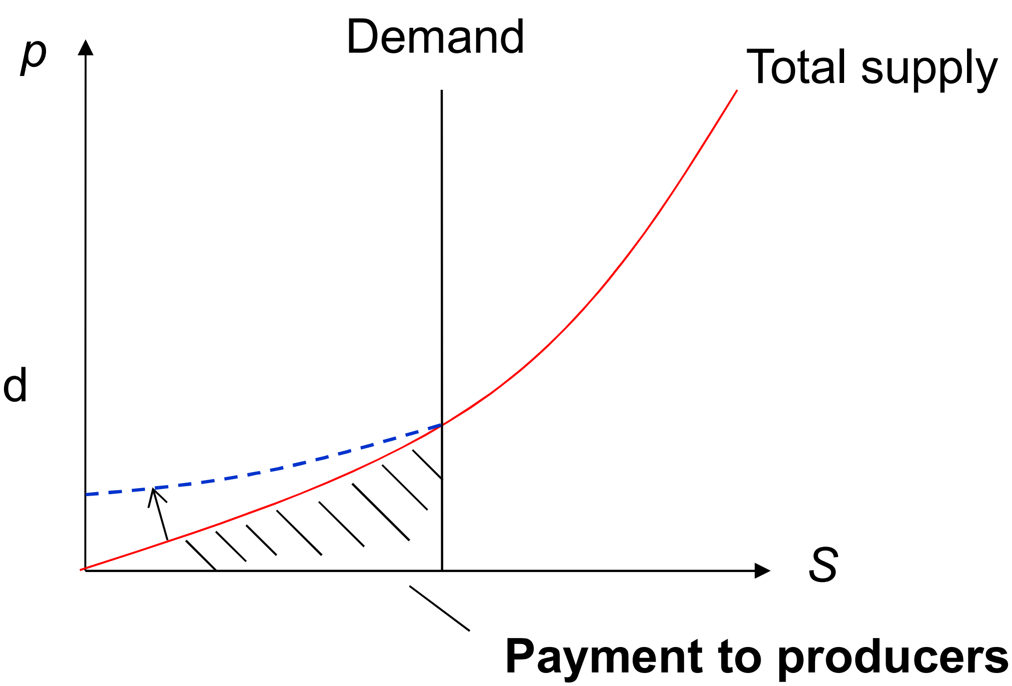
# The pay-as-bid auction

Discriminatory pricing is often used by treasuries. Used in the real-time market of UK, for counter-trading in zonal markets, and in some auctions of operating reserves.

Optimal to bid close to expected marginal bid (stop-out price).




Slope around expected stop-out price depends on uncertainties.

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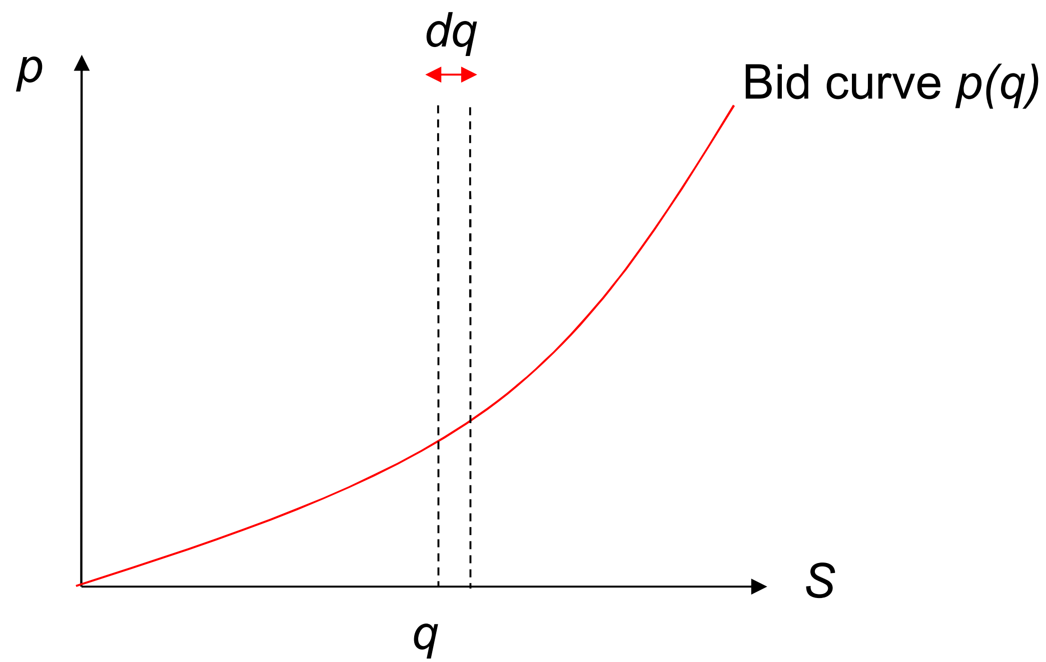
# Each producer sets many prices

With discriminatory pricing, each production plant has its own price. Plants are infinitesimally small in a continuous model. In a continuous model, each incremental output, *dq*, has its own price, *p(q)*.




The bid function *p(q)* is the inverse of the supply function $S_i(p)$.

# Solving for SFE for discriminatory pricing

The total expected profit is maximized by maximizing the expected profit density $\pi_i$ for each $q$.

$$\pi_i(p(q), q) = \big(p(q) - C_i{}'(q)\big)\big(1 - \psi(p(q), q)\big)$$

The optimal *p(q)* can be determined from (Anderson et al., 2013):

$$\frac{\partial \pi_i}{\partial p} = \big(1 - \psi(p(q), q)\big) - \big(p(q) - C_i{}'(q)\big)\frac{\partial \psi}{\partial p} = 0$$

Symmetric pure-strategy NE with N firms, inelastic demand and additive demand shocks with probability distribution *G()*:

$$\psi(p, q) = G\big(q + (N-1)S(p)\big)$$

$$\underbrace{\frac{\Big(1 - G\big(NS(p)\big)\Big)}{G'\big(NS(p)\big)}}_{Inverse\ hazard\ rate} = \Big(p - C_i{}'\big(S(p)\big)\Big)\,(N-1)S'(p)$$

ODE solution is well-behaved for decreasing hazard rates, such as Pareto distribution of the second kind.

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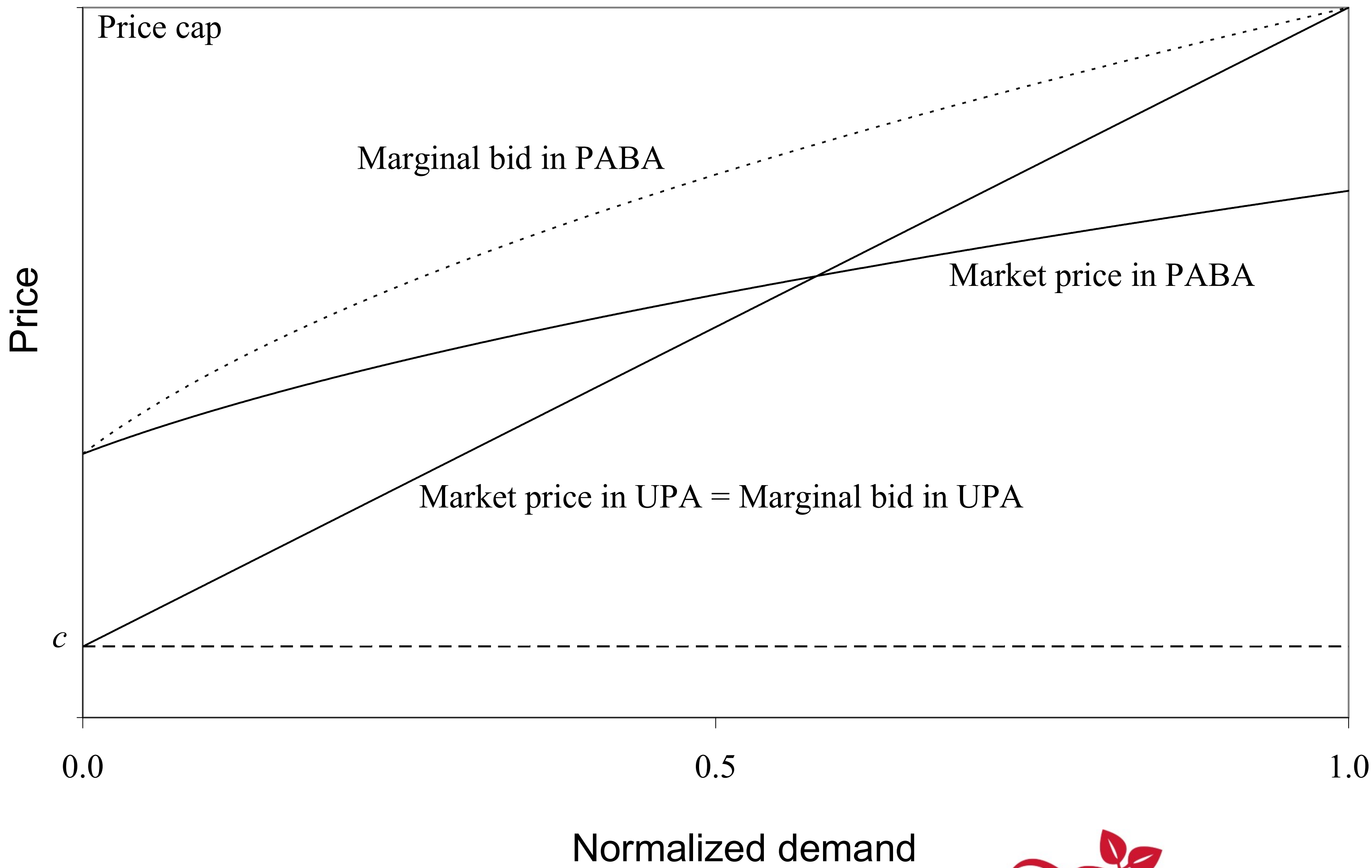
# Comparing pay-as-bid (PABA) and uniform-price auctions (UPA)




Holmberg (2009)

# Strategic bidding in electricity markets:

# Mixed-strategy NE in discriminatory auctions

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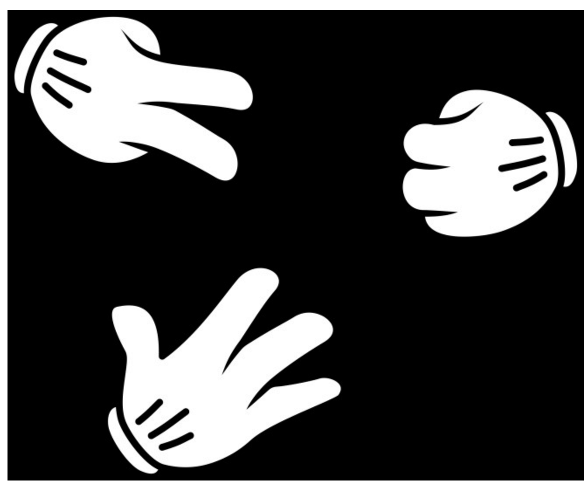
# Mixed-strategy NE in pay-as-bid auctions

A difference with discriminatory pricing is SFE are ex-ante optimal => The equilibrium depends on the probability distribution of demand shocks. Often pure-strategy SFE, where producers use deterministic strategies do not exist, especially if demand shocks follow a probability distribution with an increasing hazard rate. Often producers will instead use randomized strategies, mixed strategy NE.

In practice, this would typically mean that small variations in a producer's cost, which are only observed by the producer, would have a large impact on its offer curve. This corresponds to Harsanyi's purification theorem (Harsanyi, 1975).



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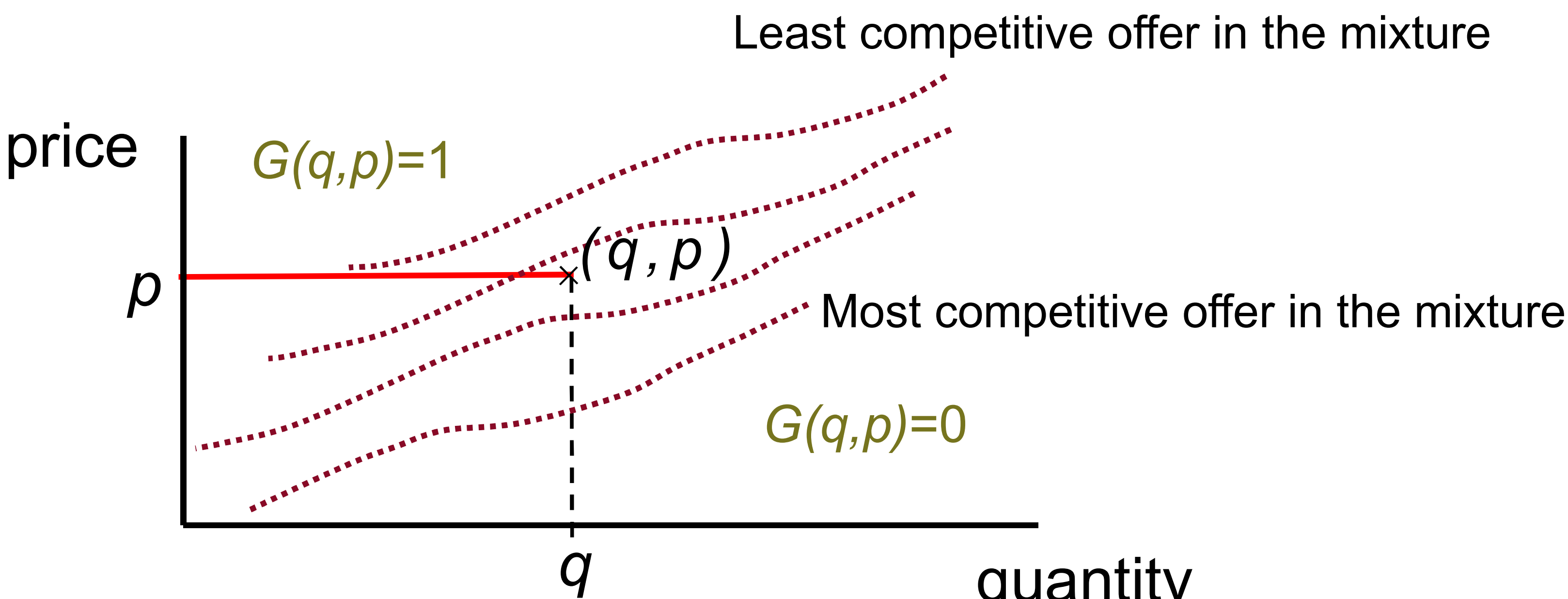
# Offer distribution function

In the general case a mixed (randomized) offer strategy can be represented by an offer distribution function, *G(q,p)*. It is the probability that the quantity *q* is offered at the price *p* (or lower).




Anderson et al. (2013)

## The market distribution function for mixed strategy NE

Assume demand has additive demand shock $\varepsilon$ and competitor uses mixed-strategy =>two sources of uncertainty in residual demand of producer. For every price *p,* the residual demand is given by difference between $D(p)+\varepsilon$ and competitor's random supply $S_j(p)$.

Probability distribution for difference of two independent random variables is cross-correlation/convolution of their individual distributions=> probability that an offer $(q_i,p)$ is rejected is:

$$\psi_i(q_i,p)=\int_{-\infty}^{\infty} f(\varepsilon)G_j(\varepsilon+D(p)-q_i,p)d\varepsilon$$

In a discriminatory auction, a Fourier transform (or similar) can be used to numerically solve for $G(q,p)$ from the first-order condition*:*

$$\left(1-\psi(p(q),q)\right)-\left(p(q)-C_i'(q)\right)\frac{\partial\psi}{\partial p}=0$$

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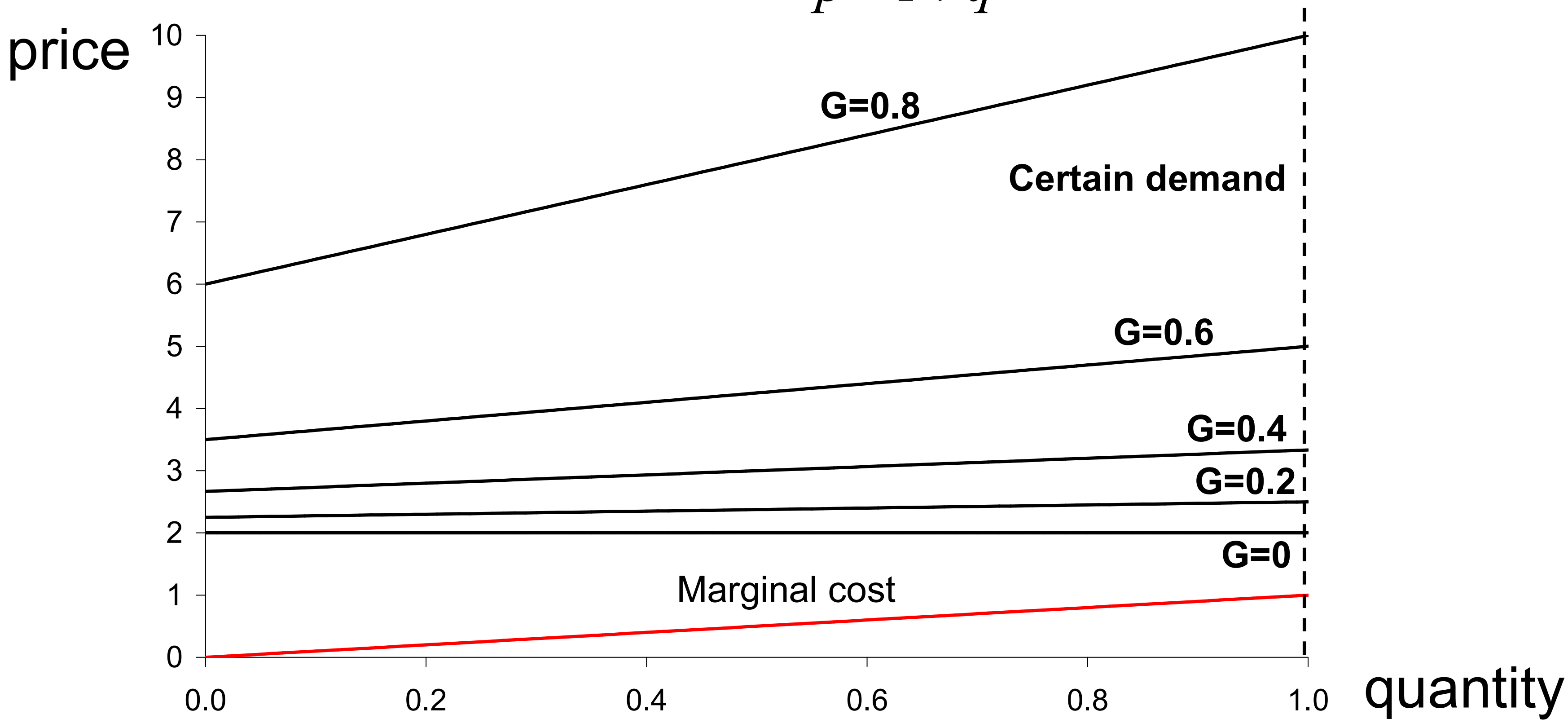
# Mixed-strategy NE for certain demand

$$G(p,q) = \frac{p - \underline{p}}{p - 1 + q}$$

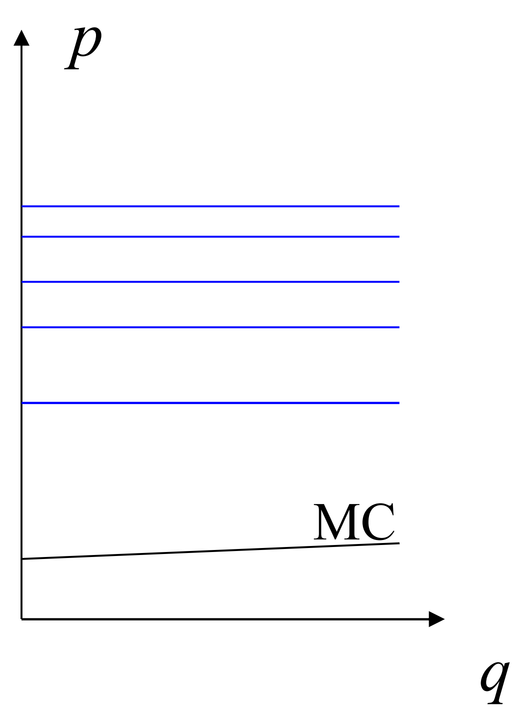



Offer distribution function *G(q,p)* provides fundamental description of mixture. It can be realized in different ways, but how is irrelevant to competitor.

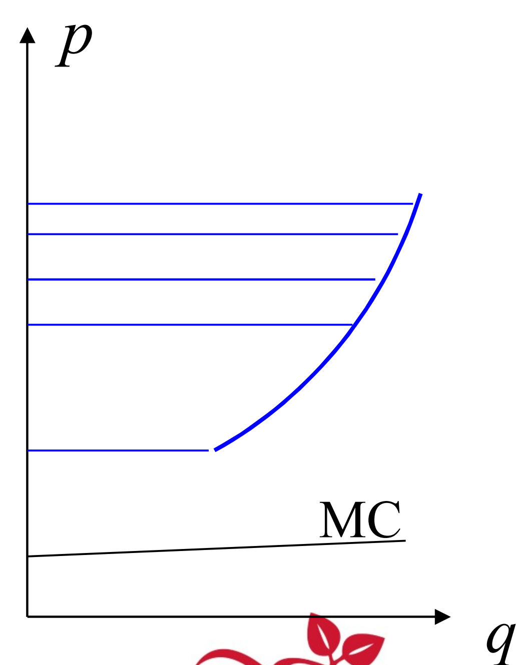
# Mixed-strategy NE over partly horizontal supply functions (binding slope constraints)

Supply functions are often restricted to be non-decreasing => There are also mixed strategy NE over partly horizontal supply functions (with binding slope-constraints) (Anderson et al., 2013)

Bertrand-Edgeworth NE




Hockey-Stick mixture:

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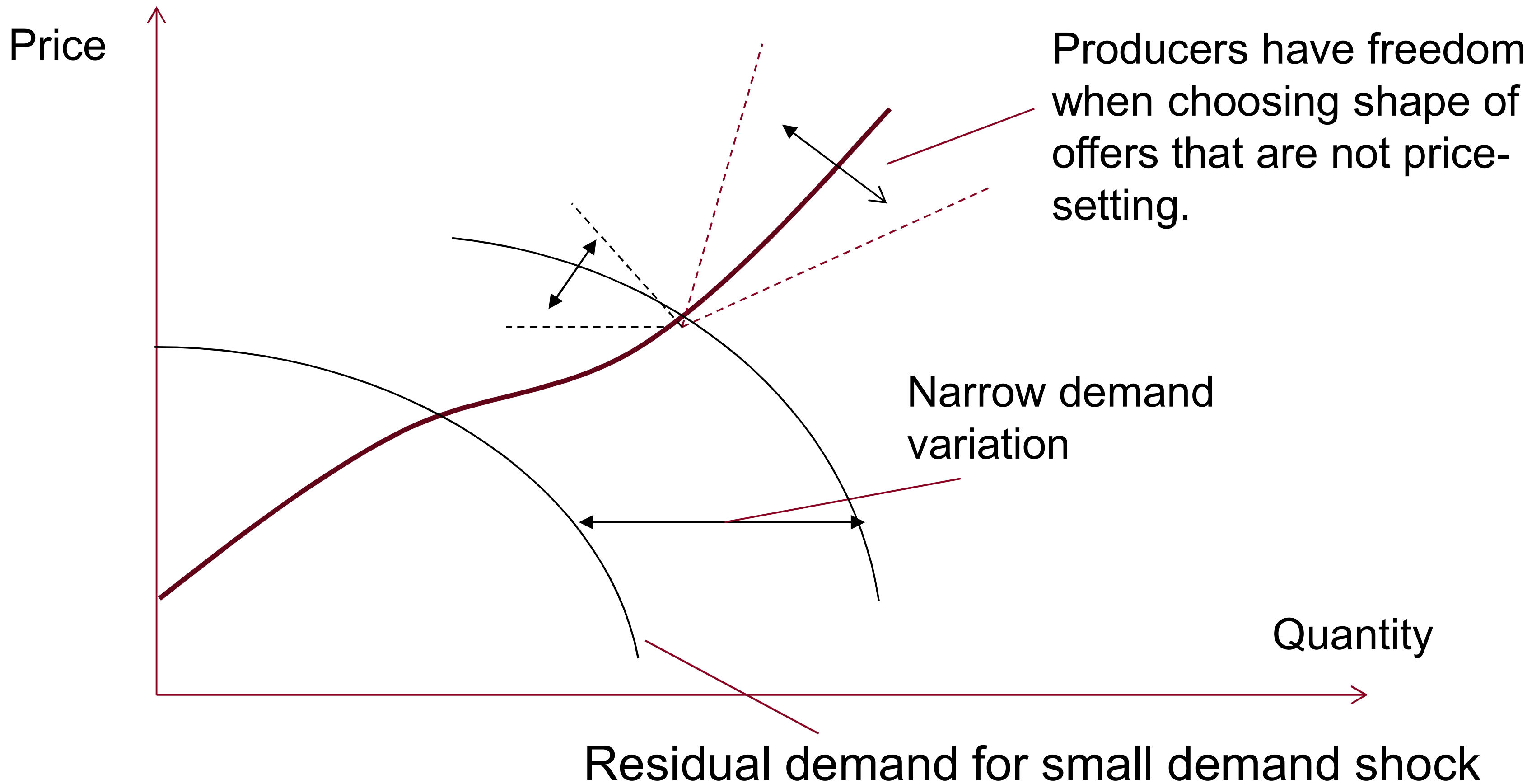
# Advantages with pay-as-bid pricing

Advantage:

- If market capacity is non-restrictive, flat bids improve competition.
- All accepted bids are price-setting => less flexibility for surely accepted bids
  - reduces risk of having multiple equilibria (avoids really bad equilibria)



# Disadvantages with pay-as-bid pricing

- Tight market capacity: flat bids lead to unpredictable price variations (price instability) (Anderson et al., 2013) => uneven and inefficient allocations.
- Multiple prices is not good when calculating a spot-price and for hedging.

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# Strategic bidding in electricity markets:

# Nodal pricing in constrained transmission networks

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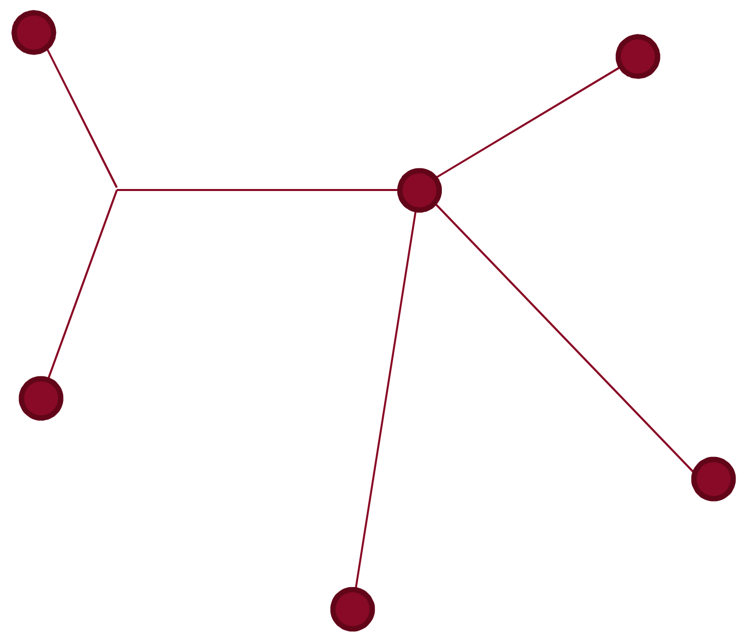
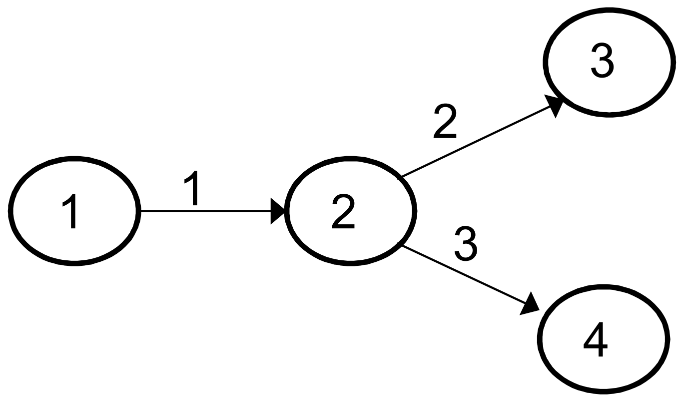
# Nodal pricing in a radial transmission network



Each node *m* has its local (nodal) market price and each line/arc *k* between nodes has a capacity constraint $\underline{t}_k \leq t_k \leq \overline{t}_k$.

# Node-arc incidence matrix

A node-arc incidence matrix **A** can be used to describe a network with rows <-> nodes and columns <-> lines.




$$\mathbf{A} = \begin{bmatrix} -1 & 0 & 0 \\ 1 & -1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

We let **t** be a column vector of flows in the lines, so **At** is net flow into each node.

# Economic dispatch problem

Producers submit supply functions to the electricity market. They are statements of costs, which may not be entirely truthful. Let $C_m(q_m)$ be the total stated production cost for producing $q_m$ units of electricity at node $m$. $\boldsymbol{\varepsilon}$ is a vector with nodal demand. The market operator chooses nodal outputs in order to minimize total production costs in the network => An economic dispatch problem (Chao and Peck, 1996; Bohn et al., 1984).

$$\min_{\boldsymbol{q}} \sum_{m=1}^{M} C_m(q_m)$$

s.t.

$$\mathbf{At} = \boldsymbol{\varepsilon} - \mathbf{q}$$

$$\underline{\mathbf{t}} \le \mathbf{t} \le \bar{\mathbf{t}}$$

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# Karush Kuhn Tucker conditions

Economic dispatch problem gives necessary and sufficient KKT conditions:

$$\mathbf{A}^{\mathbf{T}}\mathbf{p} = \boldsymbol{\rho} - \boldsymbol{\sigma}$$
$$\mathbf{A}\mathbf{t} + \mathbf{q} = \boldsymbol{\varepsilon}$$
$$\mathbf{0} \leq \boldsymbol{\rho} \perp \bar{\mathbf{t}} - \mathbf{t} \geq 0$$
$$\mathbf{0} \leq \boldsymbol{\sigma} \perp \bar{\mathbf{t}} + \mathbf{t} \geq 0$$
$$\mathbf{q} = \mathbf{S}(\mathbf{p})$$

where $\boldsymbol{\rho}$ is vector of Lagrange multipliers for flows in positive direction and $\boldsymbol{\sigma}$ is vector of Lagrange multipliers for flows in negative direction.

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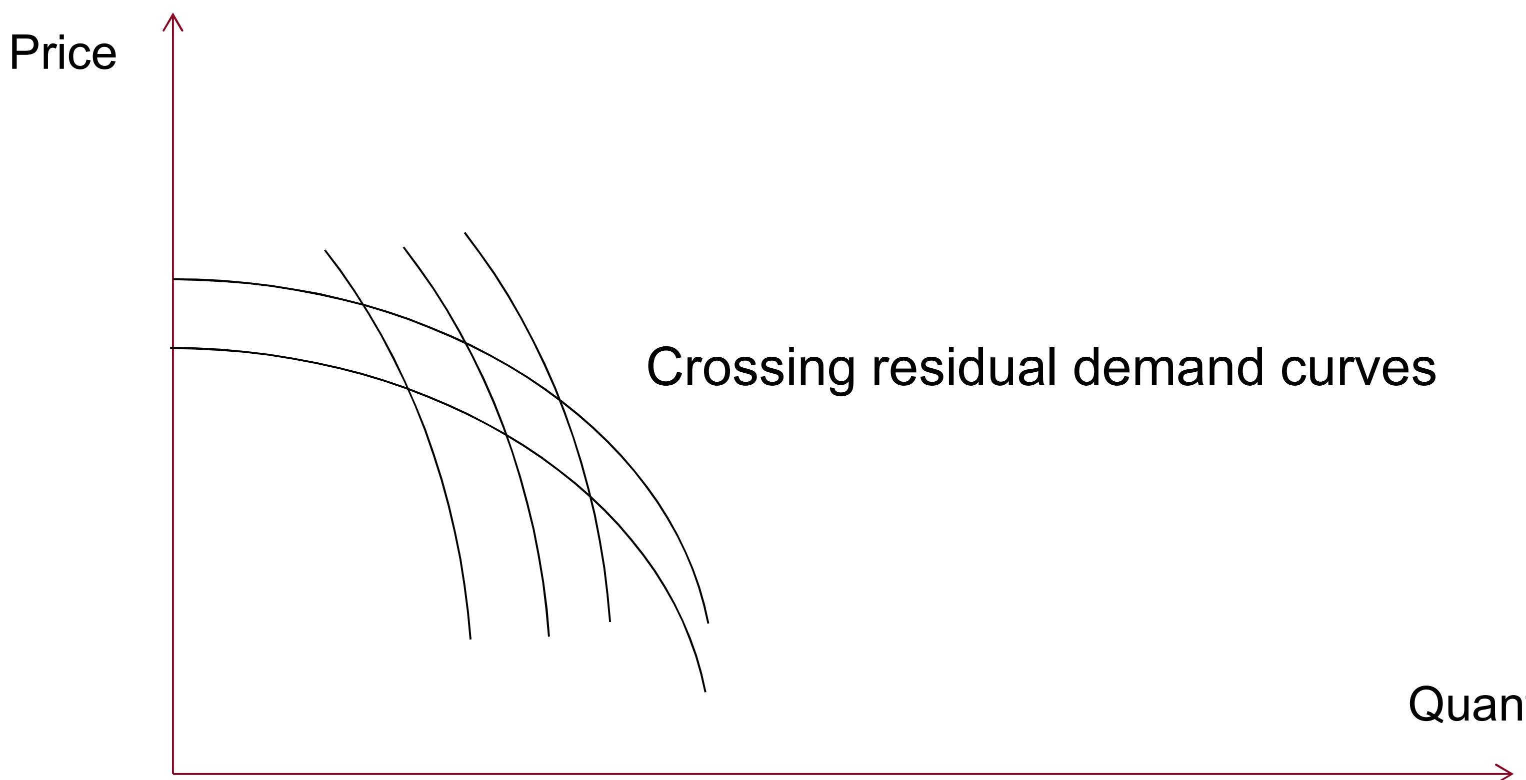
# Strategic bidding in capacity-constrained network

Problem:

1) Demand shock in each node => multiple shocks
2) Slope of residual demand depends on what lines are congested.




Good news: FOC in uniform-price auction:

$$S_i(p) = -\left(p - C_i{}'(S_i(p))\right)\frac{\partial\psi/\partial p}{\partial\psi/\partial q}$$

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# Congestion state

Each line can have three states (import constrained, export constrained and uncongested). In a network with *K* lines there are in total $3^K$ combinations of states. Holmberg and Philpott (2017) denote each of these combinations by an integer $\omega \in \Omega = \{1,2,\ldots,3^K\}$ called a *congestion state*. A similar approach is used by Wilson (2008).

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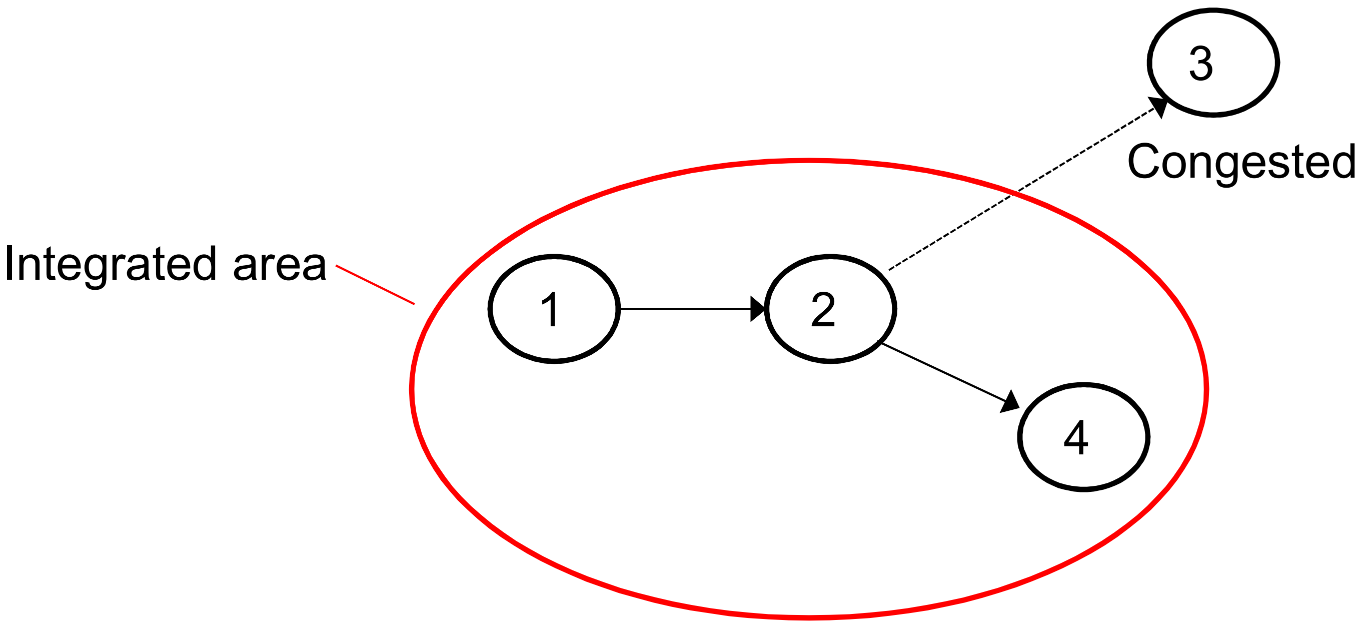
# Congestion state

As an example, we can consider a congestion state $\omega$ where exports to node 3 are congested, while all other transmission lines are uncongested.




Nodes 1,2 and 4 are integrated and can be treated as one node. Flows to node 3 are fixed, so node 3 is isolated from marginal changes in other nodes.

We use $\Xi_m(\omega)$ to denote all nodes that are integrated with node *m* in congestion state $\omega$. Example:
$\Xi_1(\omega) = \Xi_2(\omega) = \Xi_3(\omega) = \{1,2,4\}$ and $\Xi_3(\omega) = \{3\}$.

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# Slope of residual demand in congestion state

Consider a producer *n* with output *q* and price *p* in node *m*. Let $Q_{mn}(p)$ be the supply of this producer. Let $S_m(p)$ be the total nodal supply in node *m*, and let $S_{m,-n}(p) = S_m(p) - Q_{mn}(p)$.

Demand is inelastic => The slope of the residual demand of a producer in node *m* is:

$R'(p,\omega)$=$-S'_{m,-n}(p) - \sum_{j \in \Xi_m(\omega)\setminus\{m\}} S'_j(p)$.

When solving for its market distribution function, $\psi(p,q)$, we find it convenient to calculate one such probability function for each congestion state $\omega$. Congestion states are disjoint, so

$$\psi(p,q) = \sum_\omega \psi(p,q,\omega)$$

$$-\frac{\frac{\partial\psi(p,q,\omega)}{\partial p}}{\frac{\partial\psi(p,q,\omega)}{\partial q}} = R'(p,\omega)$$

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# Optimal supply function in transmission network

It follows from the first-order condition for uniform-price auctions that:

$$Q_{mn}(p) \ = -\left(p - C_i{}'(Q_{mn}(p))\right)\frac{\frac{\partial \psi}{\partial p}}{\frac{\partial \psi}{\partial q}}$$

$$= -\left(p - C_i{}'(Q_{mn}(p))\right)\sum_{\omega}\frac{\frac{\partial \psi(p,q,\omega)}{\partial p}}{\frac{\partial \psi(p,q,\omega)}{\partial q}}\frac{\frac{\partial \psi(p,q,\omega)}{\partial q}}{\frac{\partial \psi}{\partial q}}$$

$$= \left(p - C_i{}'(Q_{mn}(p))\right)\textstyle\sum_{\omega} R'(p,\omega)\,\hat{P}(\omega|p,Q_{mn}(p)),$$

where $\hat{P}(\omega|p,q)$= $\frac{\frac{\partial \psi(p,q,\omega)}{\partial q}}{\sum_{,\hat{\omega}}\frac{\partial \psi(p,q,\hat{\omega})}{\partial q}}$ is a conditional probability, the probability that the system is in the congestion state $\omega$ conditional on that the residual demand curve passes through the point $(p,q)$ =>

$$Q_{mn}(p) \ = \left(p - C_i{}'(Q_{mn}(p))\right)\mathbb{E}(R'(p,\omega)|D^{\varepsilon}(p) = Q_{mn}(p))$$

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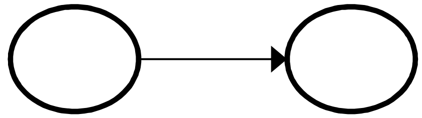
# Two-node network with symmetric producers



Market integration factor

$\mu = Expected\ number\ of\ nodes\ that\ are\ integrated\ in\ a\ symmetric\ network$

In a two-node network with symmetric firms and uniformly distributed demand shocks we have (Holmberg and Philpott, 2017):

$$\mu = \frac{4\bar{t}+N\bar{q}}{2\bar{t}+N\bar{q}},$$

where *N* is number of producers per node, $\bar{t}$ is the transmission capacity and $\bar{q}$ is the production capacity per firm.

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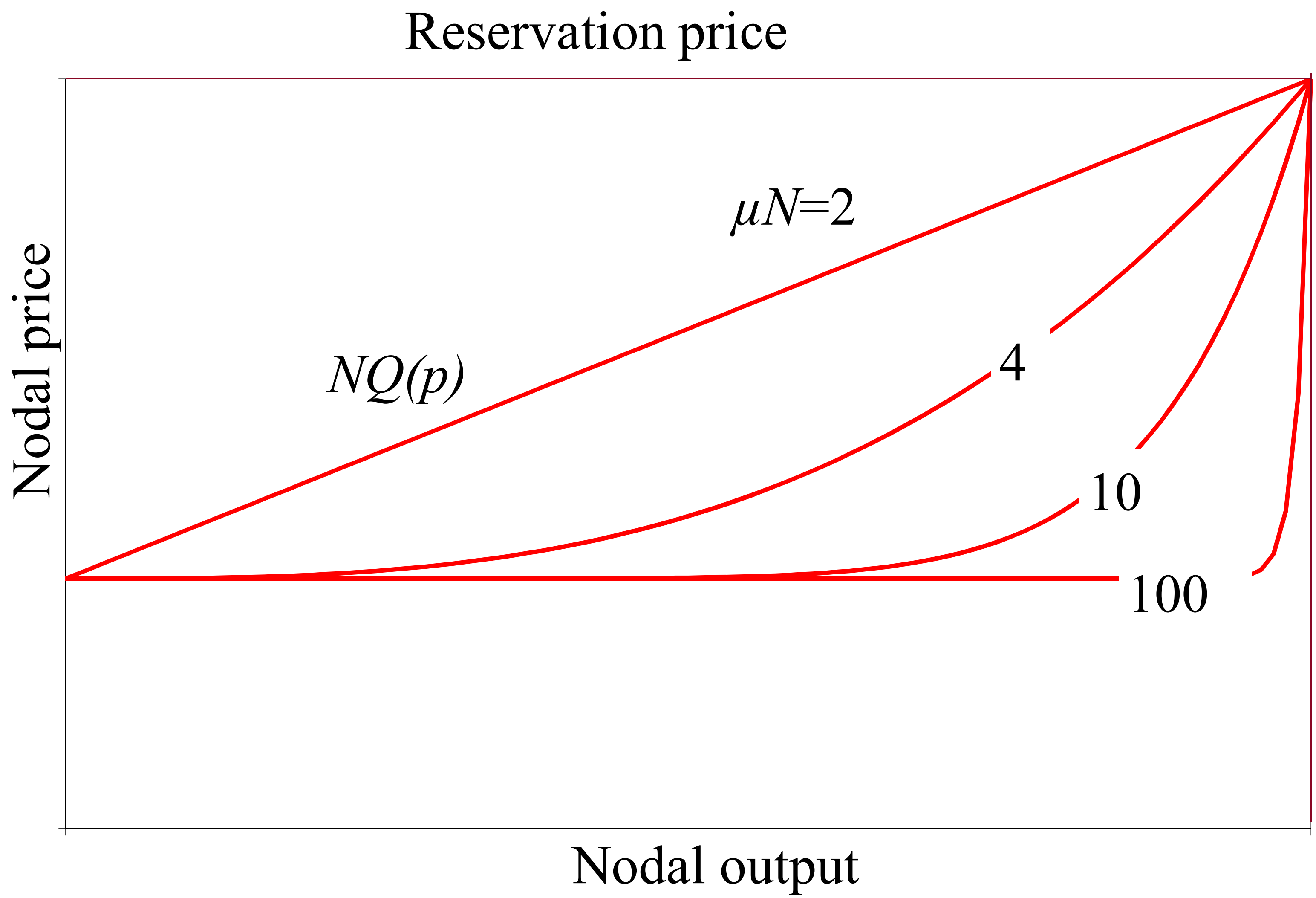
# SFE in two-node network




Ruddell (2017) simulates SFE in networks with asymmetric firms.

# Strategic bidding in electricity markets:

# Effect of discreteness/indivisibilities

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# Discreteness/indivisibilities in electricity markets

- Some production plants have to run above a minimum level when activated
- In Colombia, electricity producers must offer the whole capacity of a plant at one price/unit.
- European electricity markets have block orders that have to be entirely accepted or entirely rejected (fill or kill)
- How do such constraints influence bidding in electricity markets?

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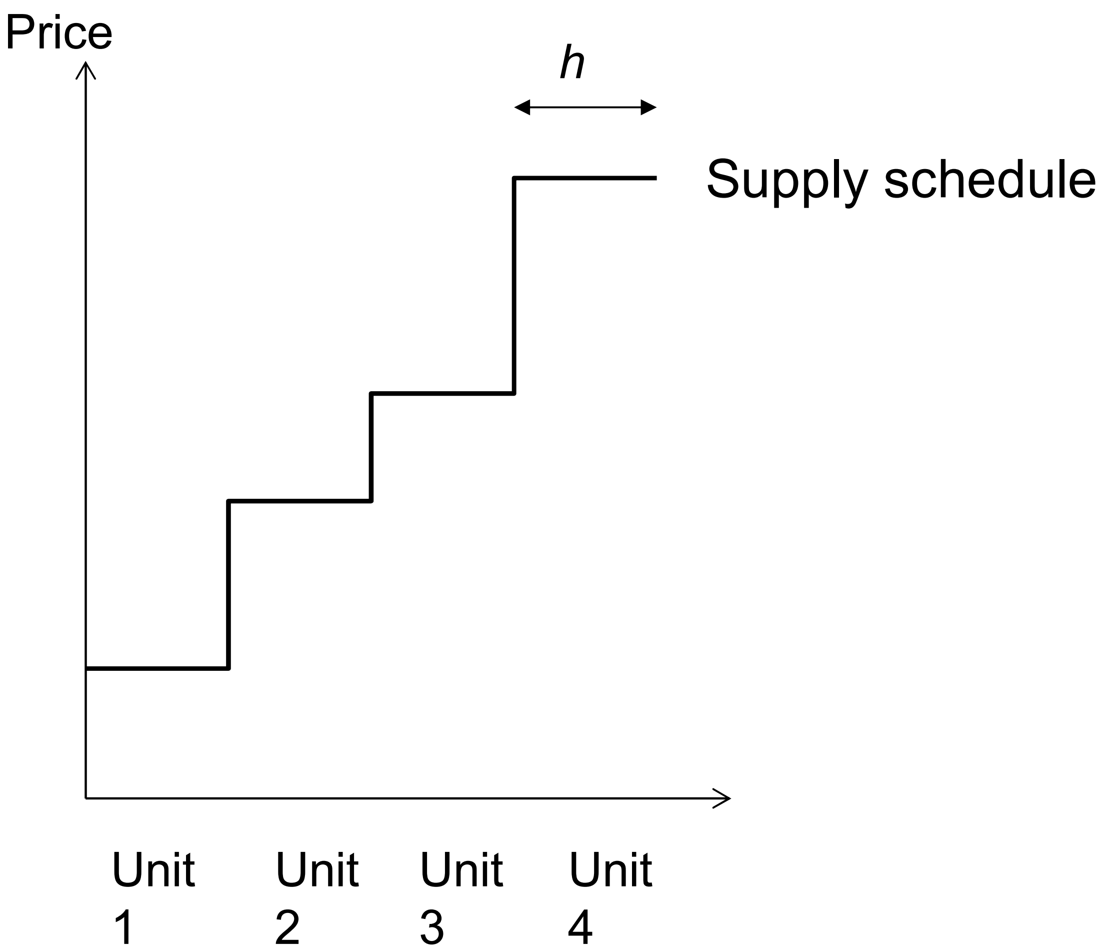
# Auction of multiple indivisible units

Each producer has multiple indivisible units and offers each unit at a different price (Anderson and Holmberg, 2015).

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# Private information

Asymmetric information: A producer *i* is assumed to know its own cost, which is unknown to the competitor. The private information is represented by the signal $\alpha_i$, which is assumed to be uniformly distributed on [0,1].

The marginal cost of plant *n* of producer *i* is: $c_n(\alpha_i)$.

Producer *i* offers plant *n* at $p_n$.

It's offer strategy can be represented by a discrete offer distribution function $G_i(p, n)$, the probability that producer *i* offers unit *n* at price *p* or lower.

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# Discrete market distribution function

$\Psi_i(n,p) =$ Probability that unit $n$ of producer $i$ is rejected if offered at $p$.

Probability distribution for difference of two independent discrete random variables is discrete cross-correlation/convolution of their individual distributions => probability that an offer *$(q_i,p)$* is rejected is:

$$\Psi_i(n,p) = \sum_{m=0}^{N} \big(G_j(p,m) - G_j(p,m+1)\big) F\big((n+m-1)h\big)$$

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# Discrete first-order condition

First-order condition (Anderson and Holmberg, 2015):

$$\frac{\partial \Pi}{\partial p_n} = \underbrace{nh(\Psi_i(n+1, p_n) - \Psi_i(n, p_n))}_{Price\ effect} - \underbrace{\frac{\partial \Psi_i(n, p_n)}{\partial p_n} h(p_n - : c_n(\alpha_i))}_{Quantity\ effect} = 0$$

Increasing $p_n$ only matters for outcomes where this price is price-setting (last accepted offer). $\Psi_i(n+1, p_n) - \Psi_i(n, p_n)$ is the probability that $p_n$ is price-setting.

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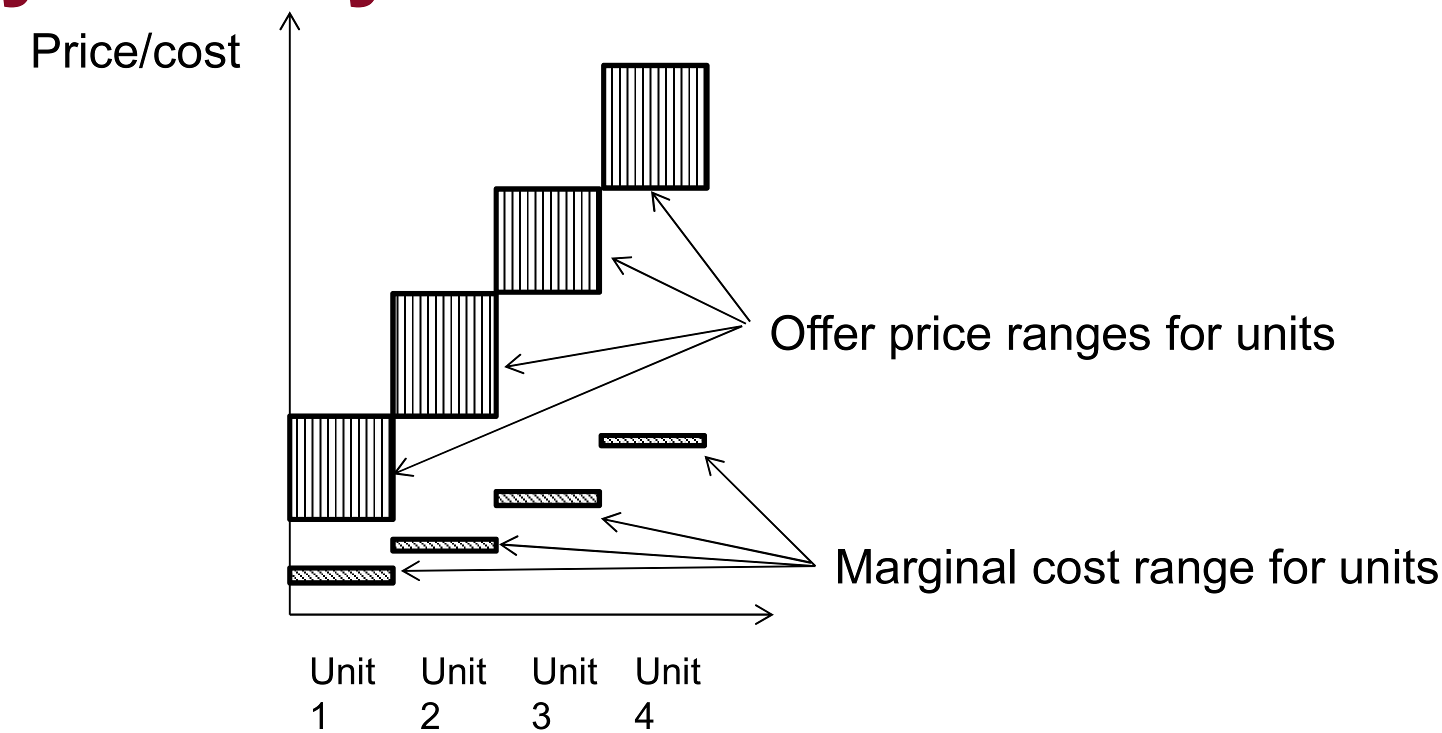
# Step separation without gaps and symmetry




Two conditions that necessarily result in step separation without gaps and symmetry are (Anderson and Holmberg, 2015):

1) Marginal cost range of units do not overlap
2) Demand is sufficiently evenly distributed:

$$3m|f(mh)-f((m-1)h)| < f((m-1)h)$$

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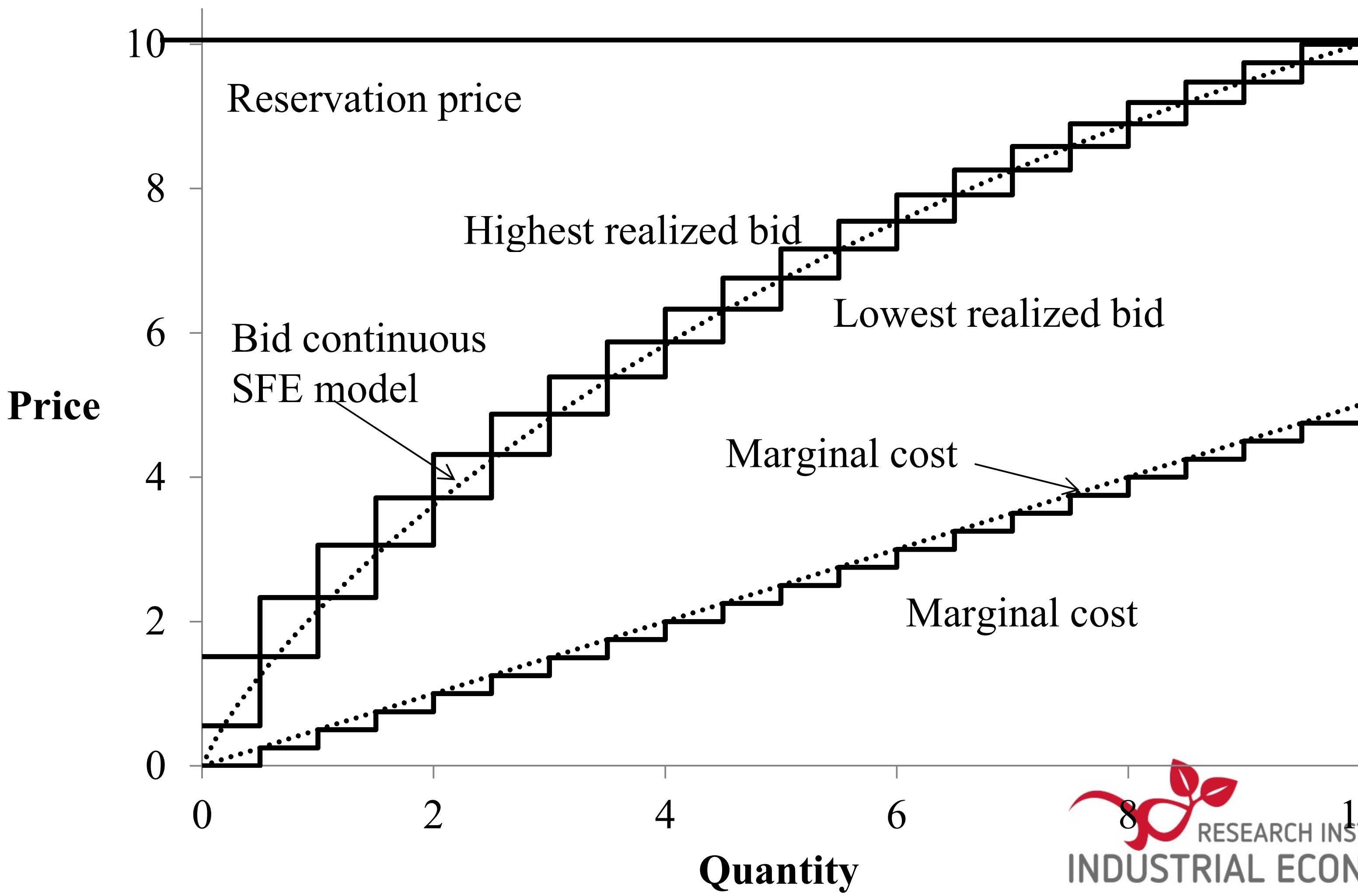
# Convergence to continuous model

In equilibrium, private signal $\alpha_i$ influences offers, even if signal has negligible influence on costs. Offers are random as for mixed-strategy NE. This adds a noise to offers. Otherwise they are similar to continuous model.

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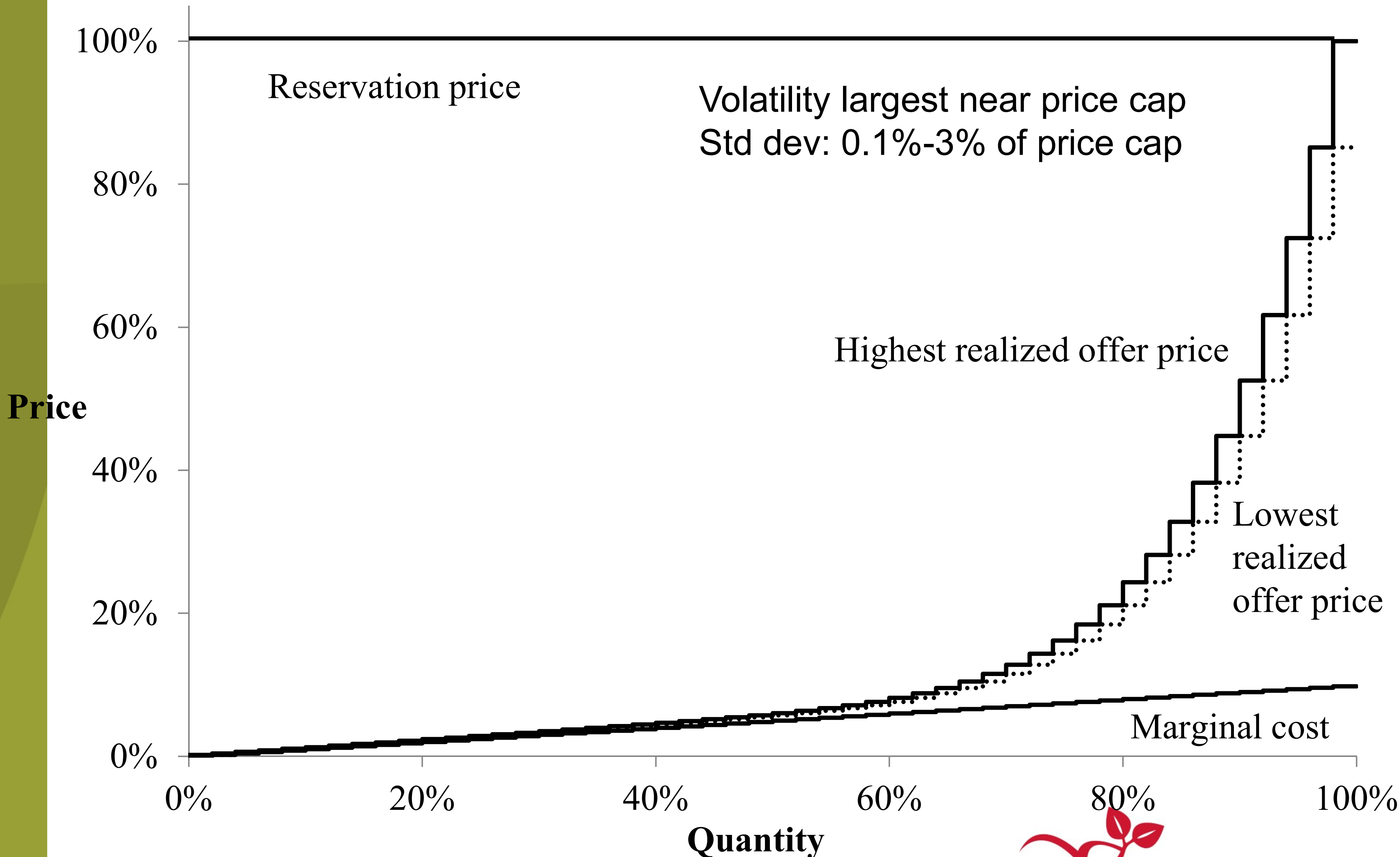
# Example: Price instability in electricity market

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