Gaussian random permutation and the free Bose gas

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We construct an infinite volume spatial random permutation (X, σ), where $\mathsf{X} \subset \mathbb{R}^d$ is a point process and $\sigma : \mathsf{X} \to \mathsf{X}$ is a permutation (bijection), associated to the formal Hamiltonian

$$H(\mathsf{X}, \sigma) = \sum_{x \in \mathsf{X}} \|x - \sigma(x)\|^2,$$

proposed by Feynman (1953) for the free Bose gas. The measures are parametrized by the density ρ of points. Bose-Einstein condensation occurs at a critical point density ρ_c, meaning that infinite cycles of the permutation should appear above ρ_c. We show that for $\rho \leqslant \rho_c$, the unique Gibbs measure for the specification induced by H is a Gaussian loop soup. This is a Poisson process of finite random walk loops with Gaussian increments, analogous to the Brownian loop soup of Lawler and Werner (2004).

We also consider Gaussian random interlacements, a Poisson process of double-infinite trajectories of random walks with Gaussian increments analogous to the Brownian random interlacements of Sznitman (2010). For $d \geqslant 3$ and $\rho > \rho_c$ the Gibbs measure is a the superposition of independent realizations of the Gaussian loop soup at density ρ_c and the Gaussian random interlacements at density $\rho - \rho_c$. The resulting measure is called Gaussian random permutation at density ρ.

We show that the point marginal of the Gaussian random permutation is the boson point process introduced by Macchi (1975) in the subcritical case and by Tamura-Ito (2007) in the supercritical case. The boson point process is permanental for a kernel related to the Green function of the Gaussian random walk.

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