A General "Power-of-d" Dispatching Framework for Heterogeneous Systems

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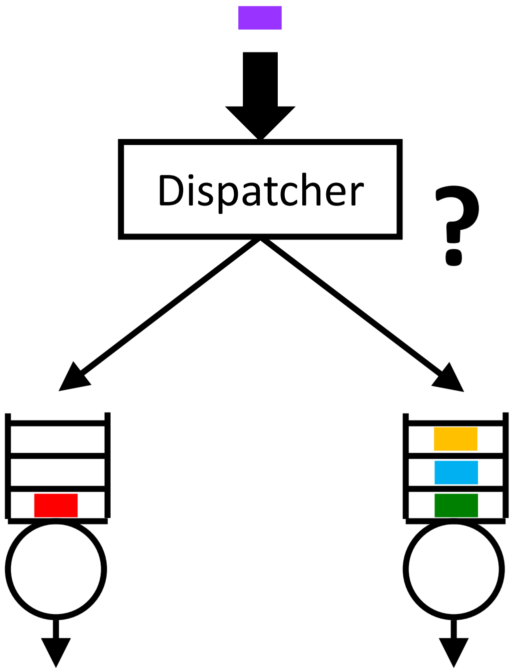
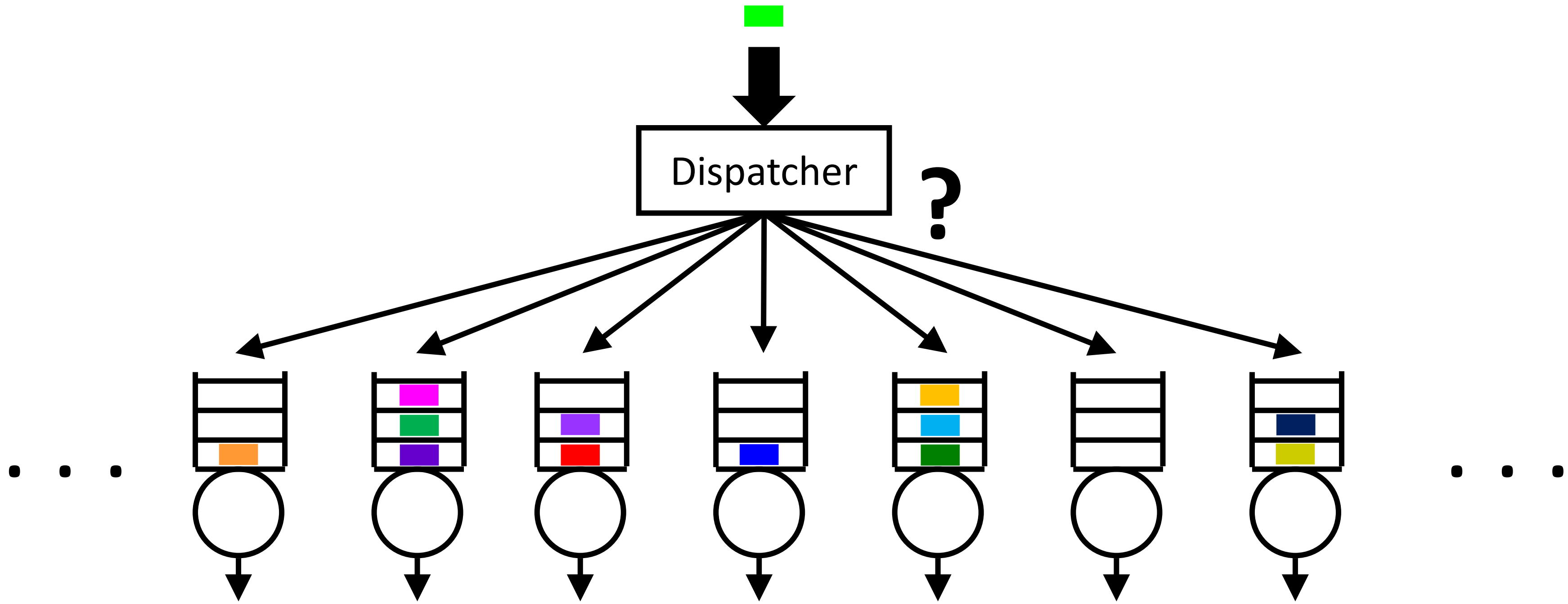
How to dispatch in multi-server systems?



Join the Shortest Queue (JSQ)

How to dispatch in multi-server systems?

large-scale

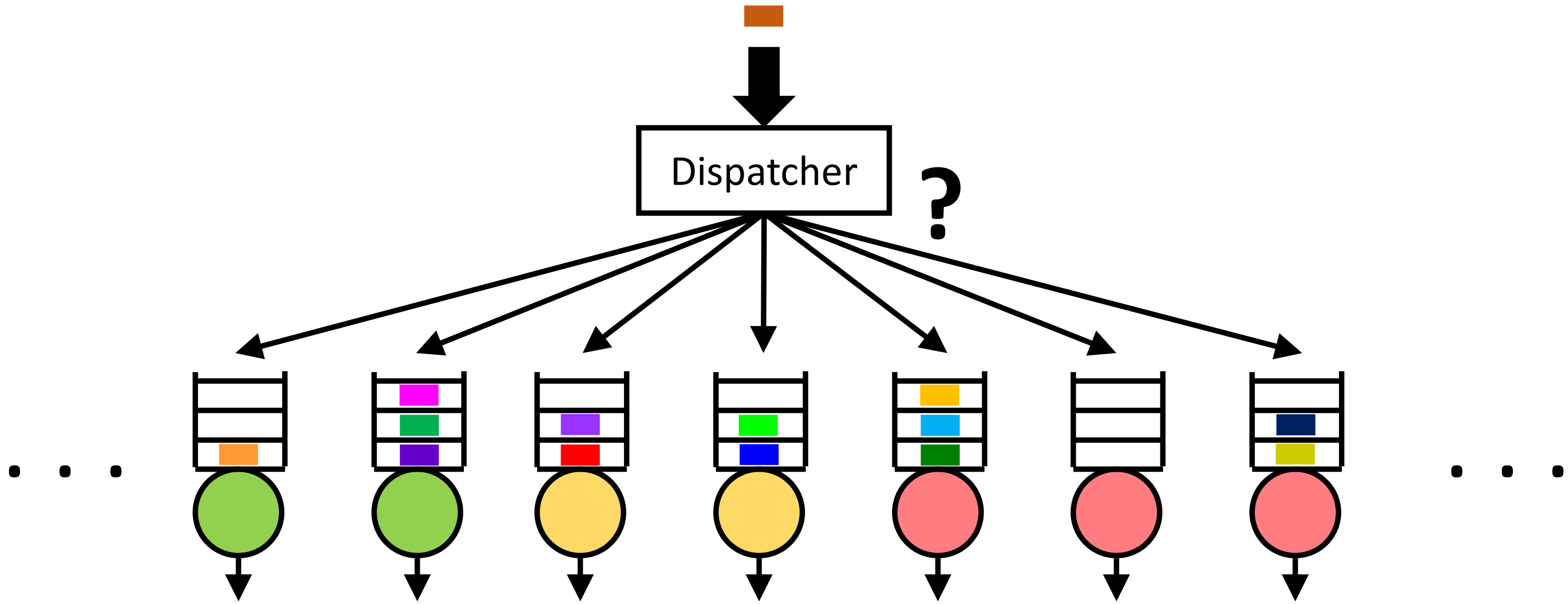


Join the Shortest Queue among d (JSQ-d)

How to dispatch in multi-server systems?

large-scale

heterogeneous



How to dispatch in multi-server systems?

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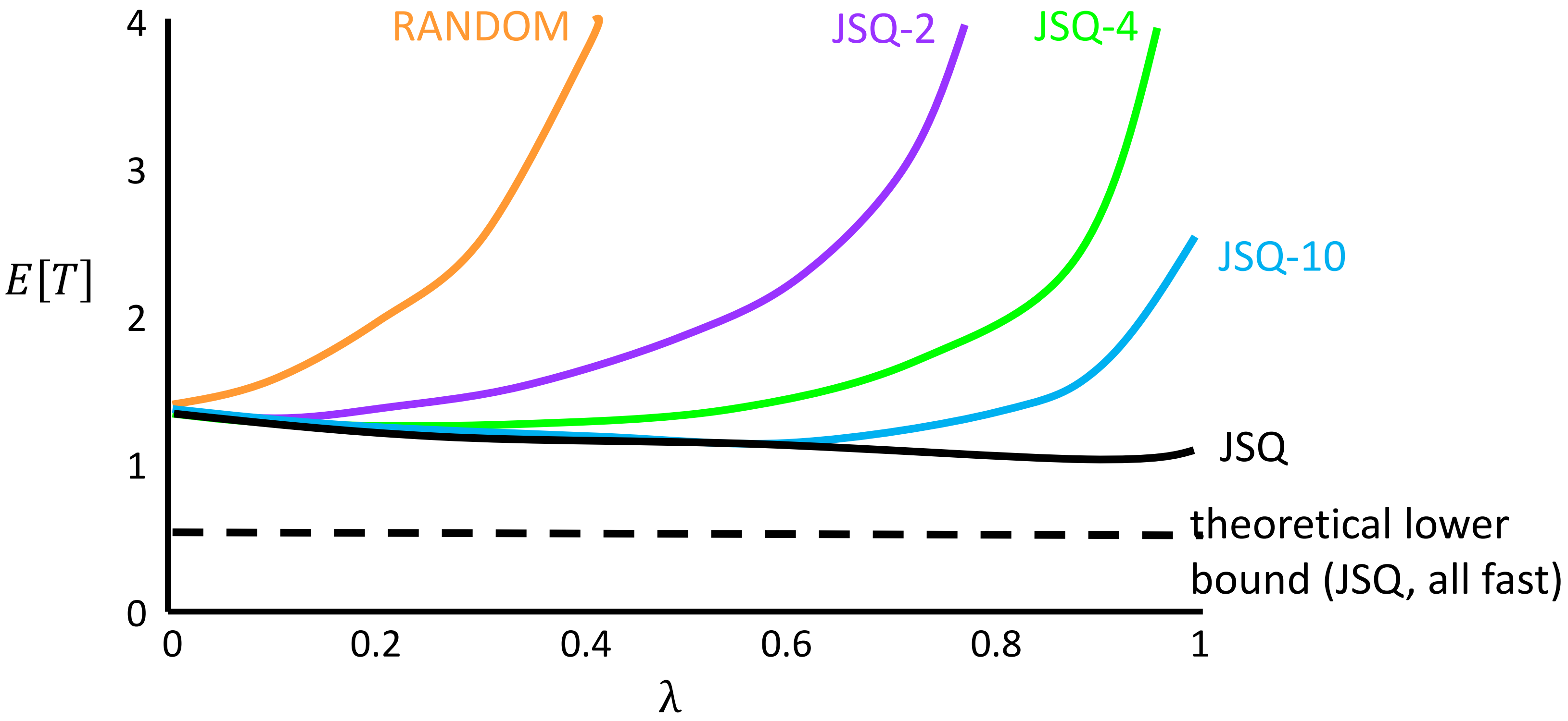
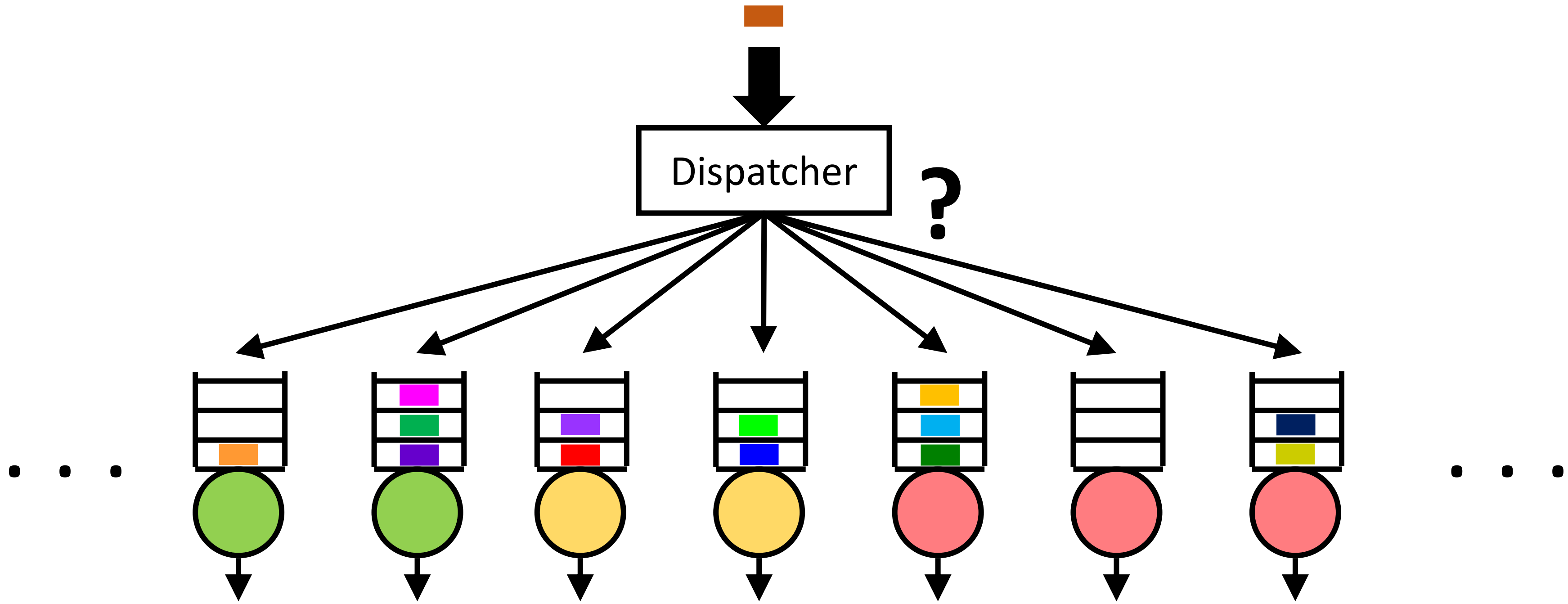


300 fast servers, rate 1.95 jobs/sec
700 slow servers, rate 0.6 jobs/sec

How to dispatch in multi-server systems?

large-scale | heterogeneous

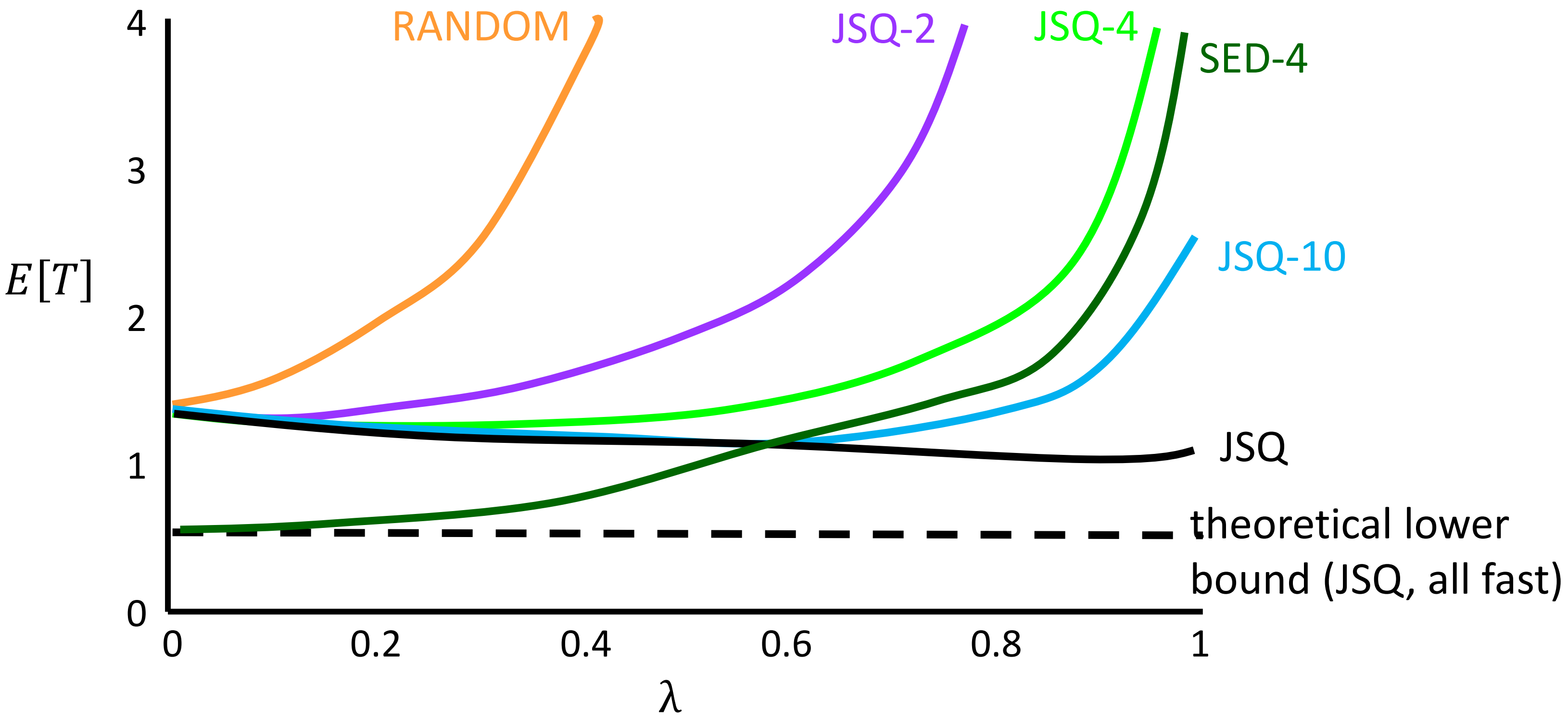


Shortest Expected Delay among d (SED-d)

How to dispatch in multi-server systems?

large-scale

heterogeneous



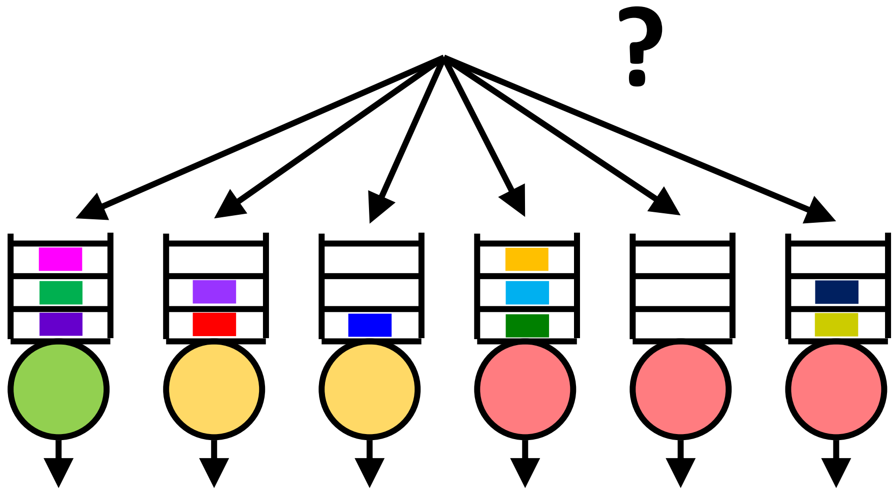
300 fast servers, rate 1.95 jobs/sec
700 slow servers, rate 0.6 jobs/sec

Leveraging Server Heterogeneity

There are **two** points at which the dispatcher can use server speed information

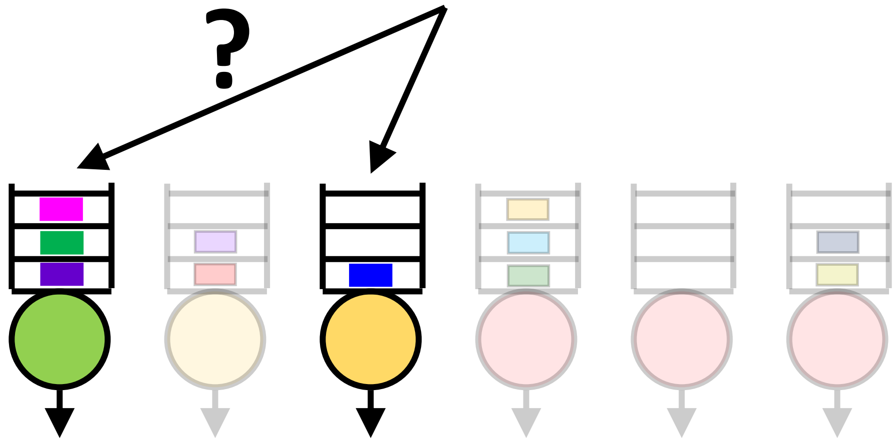
Querying Rule

Which d servers to query?

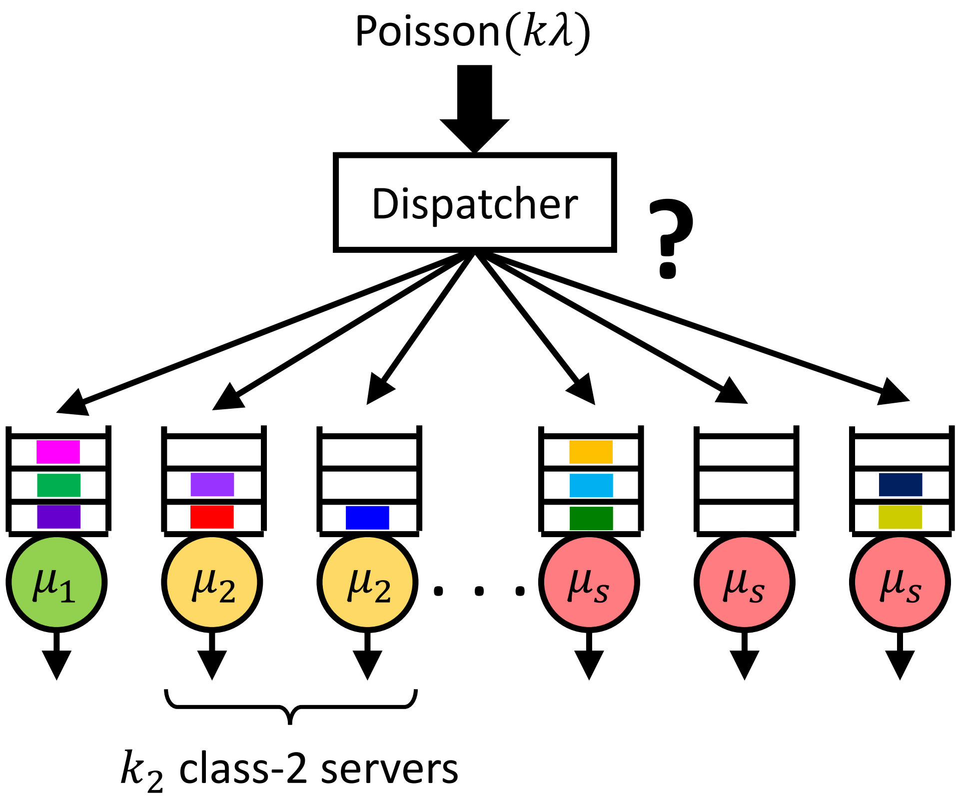


Assignment Rule

After querying, where to send the job?



System Model



Modeling Assumptions

k servers, s server speed classes $k_i = kq_i$ servers in class i

Independent service times, distribution G_i with mean $1/\mu_i$

$\mu_1 > \mu_2 > \cdots > \mu_s$

Poisson arrival process with rate $k\lambda$

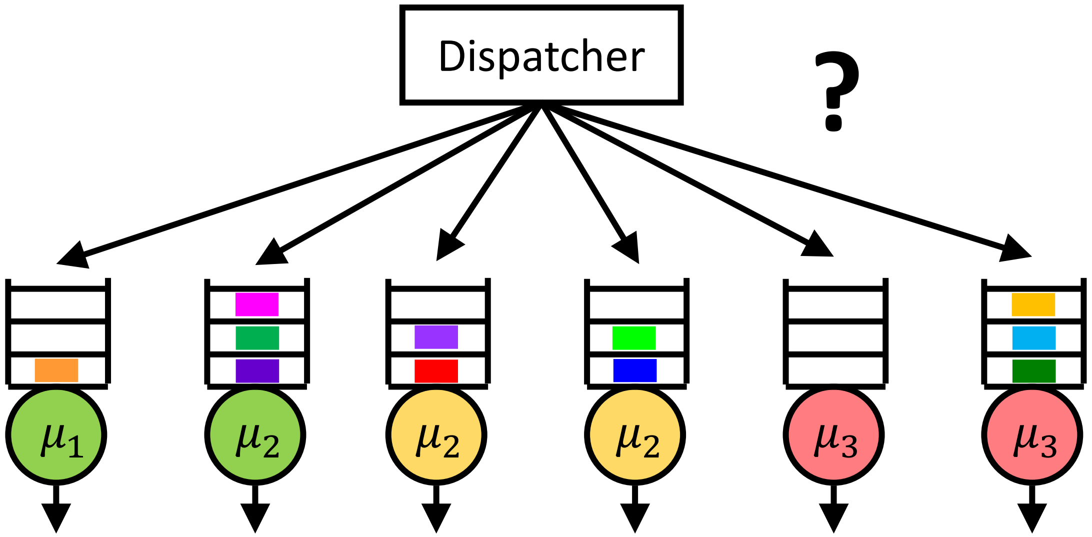
Dispatcher decides immediately to which server to send a job

Scheduling: any work-conserving policy

Goal: design dispatching policies to achieve low mean response time

Key idea: Dispatching policy = Querying rule + Assignment rule

Querying Rules



Notation

d: # of servers queried (constant, $\ll k$)

d_i: # of class-i servers queried

$\vec{\boldsymbol{d}} = (d_1, \dots, d_s)$: query mix

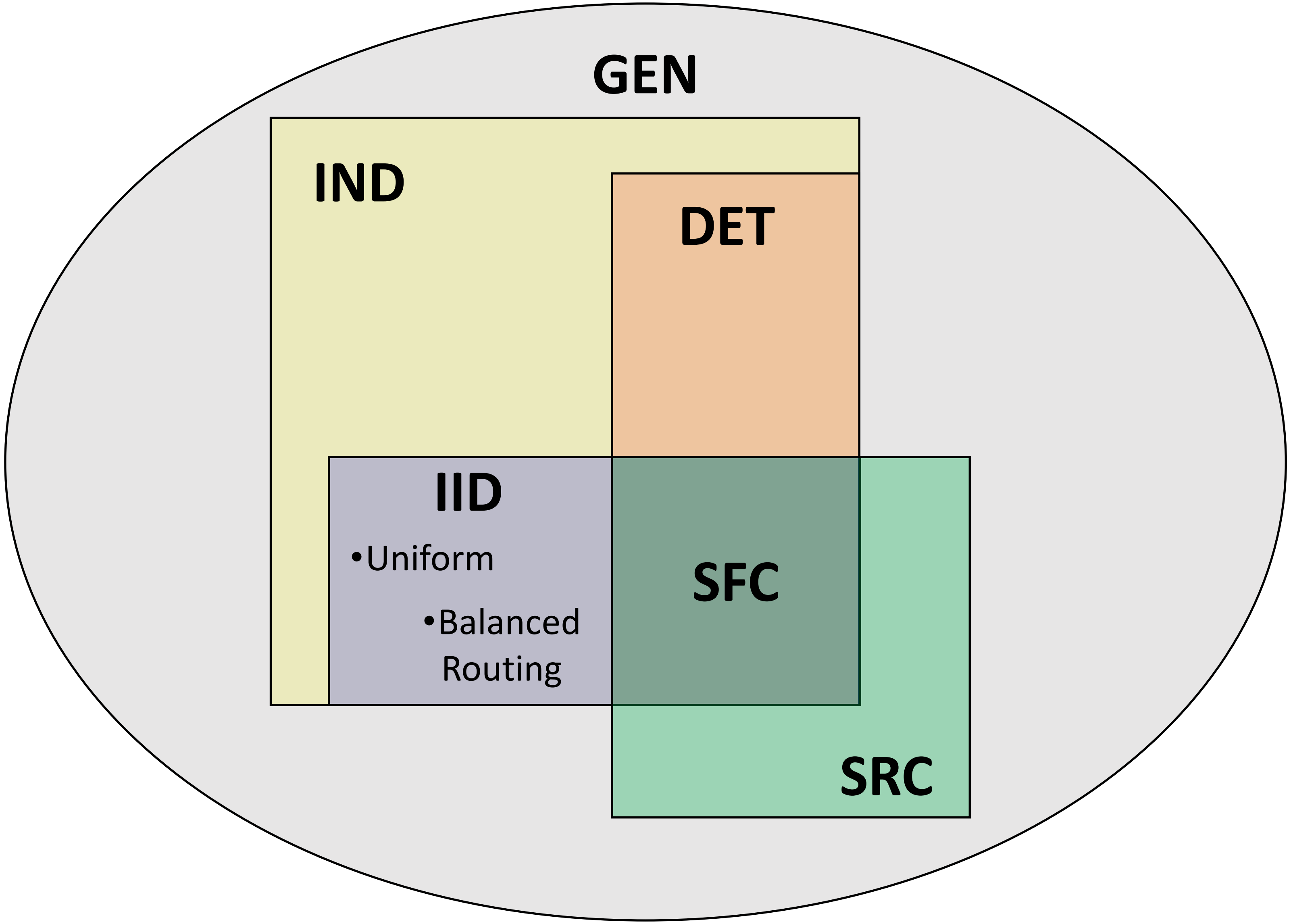
$\mathcal{D} = \{\vec{\boldsymbol{d}}: d_1 + \cdots + d_s = d\}$: set of possible query mixes

Two important properties:

- **Symmetric**: servers of the same class are treated *ex ante* identically
- **Static**: dispatcher doesn't maintain any history

Querying rule: $p(\cdot): \mathcal{D} \to [0,1]$

Querying Rules



GEN: any distribution over $\mathcal{D}$

IND: each of the d queried server classes is chosen independently

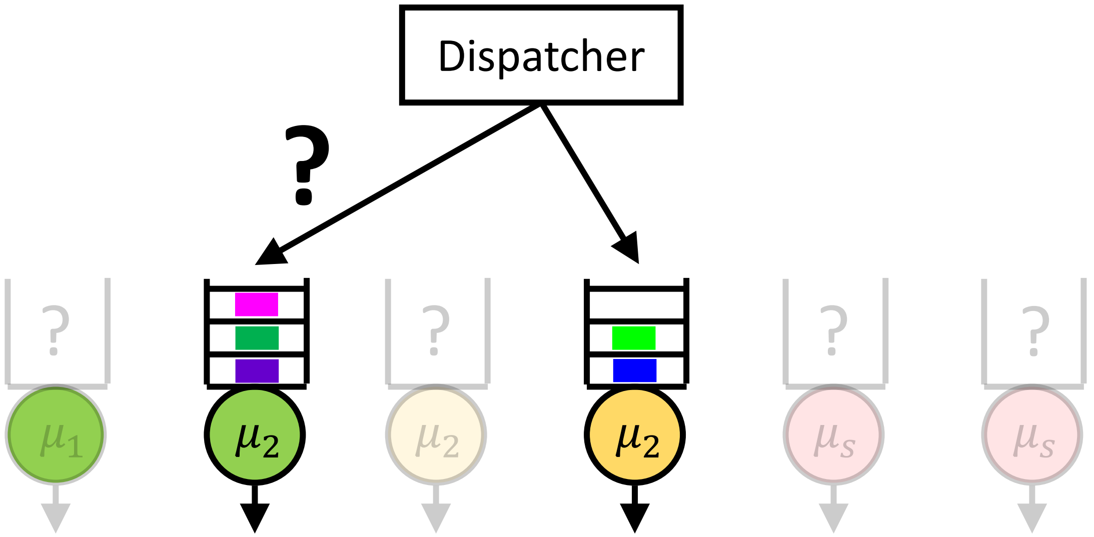
IID: each of the d queried server classes is chosen independently *from the same distribution*

DET: the same query mix is always used ($\vec{\boldsymbol{d}}$ is deterministic)

SRC: probabilistically chooses one server class, then queries d servers from that class

SFC: always queries d class-i servers for some fixed class i

Assignment Rules



Notation

$\vec{\boldsymbol{d}} = (d_1, \ldots, d_s)$: query mix

$\mathcal{D} = \{\vec{\boldsymbol{d}}: d_1 + \cdots + d_s = d\}$: set of possible query mixes

$\vec{\boldsymbol{x}}(\vec{\boldsymbol{d}})$: information obtained about the d queried servers

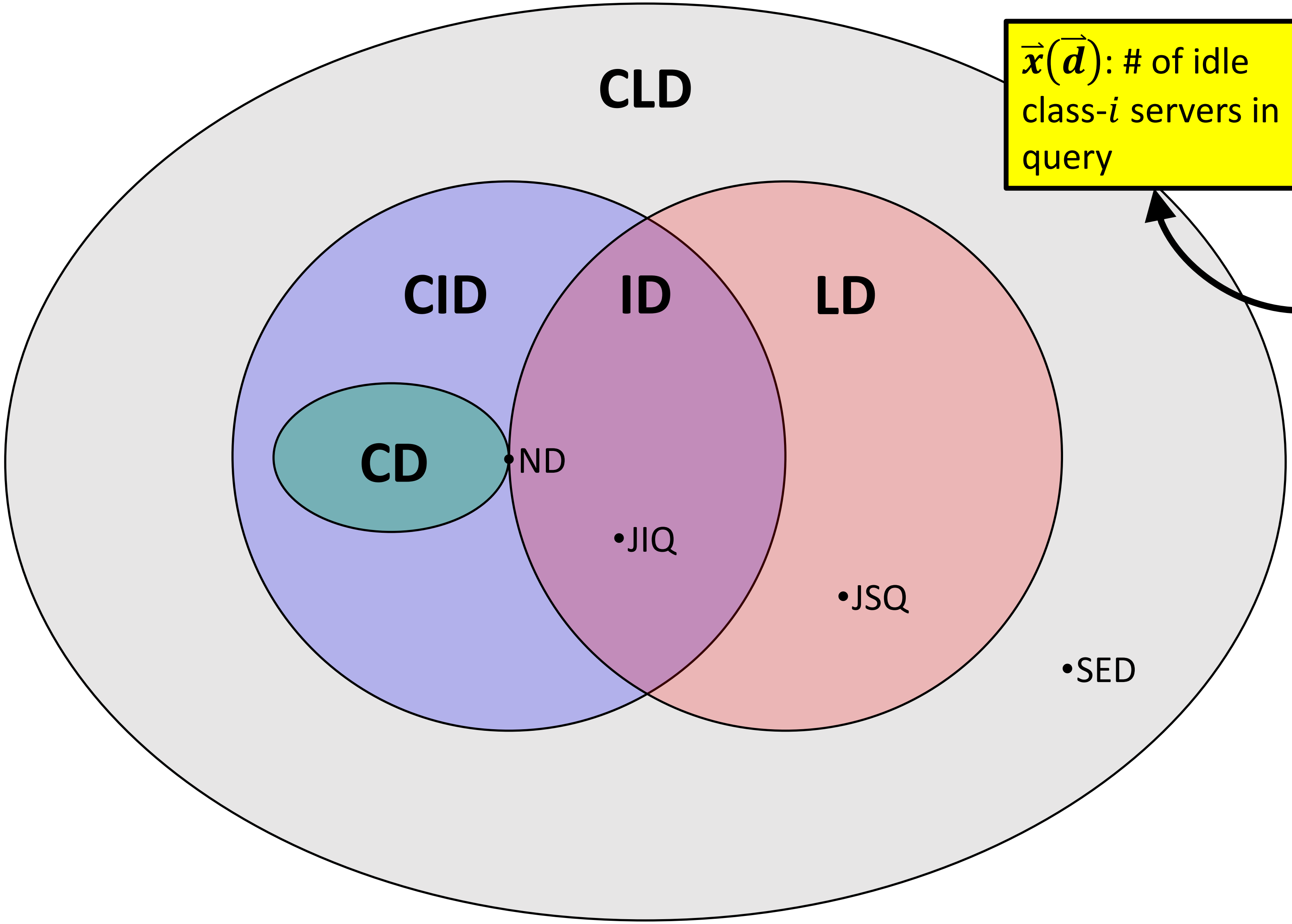
$\mathcal{X} = \{\vec{\boldsymbol{x}}(\vec{\boldsymbol{d}}): \vec{\boldsymbol{d}} \in \mathcal{D}\}$: set of possible query results

Two important properties:

- **Symmetric**: servers with the same state are treated identically
- **Static**: dispatcher doesn't maintain any history

Assignment rule: $\alpha(\cdot,\cdot): \mathcal{D} \times \mathcal{X} \to [0,1]$

Assignment Rules



$\vec{x}(\vec{d})$: # of idle class-i servers in query

CLD: can use class and queue-length information

CID: can use class and idleness information

LD: can use *only* queue-length information

ID: can use *only* idleness information

CD: can use *only* class information

Analyzing ⟨**GEN**, **CID**⟩: Approach

Goal: Find $E[T]$ given querying and assignment rules

<u>Assumptions</u>:

- $k \to \infty$, with q_i fixed
- States of all servers of the same class evolve stochastically identically
- All server states are independent

<u>Main analytic technique</u>: tag a class-i server and examine what happens whenever a new job arrives in steady-state, via mean-field analysis:

- Is the tagged server queried?
- If so, does it get the job?

Analyzing ⟨**GEN**, **CID**⟩: Preliminaries

Observation: tagged class-i server has different arrival rates when **idle** vs **busy**

- When **idle**: Poisson$\left(\lambda_i^I\right)$
- When **busy**: Poisson$\left(\lambda_i^B\right)$

Observation: once busy, server behaves like an $M/G_i/1$ with arrival rate λ_i^B

Mean busy period duration:

$$E[B_i] = \frac{1}{\mu_i - \lambda_i^B}$$

Fraction of time a server is busy:

$$\boxed{\rho_i = \frac{E[B_i]}{1/\lambda_i^I + E[B_i]} = \frac{\lambda_i^I}{\mu_i - \lambda_i^B + \lambda_i^I}}$$

Analyzing ⟨**GEN, CID**⟩: $E[T]$

Condition on the server's class:

$$E[T] = \sum_{i=1}^{s} \left(\frac{k_i(1-\rho_i)\lambda_i^I + k_i \rho_i \lambda_i^B}{k\lambda} \right) E[T_i]$$

Mean response time at a class-i $M/G_i/1$ with arrival rate λ_i^B

Any work-conserving scheduling policy!

$$= \sum_{i=1}^{s} q_i \left(\frac{(1-\rho_i)\lambda_i^I + \rho_i \lambda_i^B}{\lambda} \right) E[T_i]$$

All that remains: finding λ_i^I and λ_i^B

Analyzing $\langle \mathbf{GEN}, \mathbf{CID} \rangle$: λ_i^I and λ_i^B

$\vec{\boldsymbol{d}} = (d_1, \dots, d_s)$: query mix

Given $\vec{\boldsymbol{d}}$, probability that the tagged server is in the query: $\frac{d_i}{k_i}$

Rate at which the tagged server is queried: $\lambda k \sum_{\vec{\boldsymbol{d}} \in \mathcal{D}} p(\vec{\boldsymbol{d}}) \frac{d_i}{k_i} = \frac{\lambda}{q_i} \sum_{\vec{\boldsymbol{d}} \in \mathcal{D}} p(\vec{\boldsymbol{d}})\, d_i$

Define $r_i^I(\vec{\boldsymbol{d}})$: prob. job is sent to the tagged server, given $\vec{\boldsymbol{d}}$ and tagged server is **idle**

$r_i^B(\vec{\boldsymbol{d}})$: analogous, tagged server is **busy**

$$\boxed{\lambda_i^I = \frac{\lambda}{q_i} \sum_{\vec{\boldsymbol{d}} \in \mathcal{D}} p(\vec{\boldsymbol{d}})\, d_i r_i^I(\vec{\boldsymbol{d}})}$$

$$\boxed{\lambda_i^B = \frac{\lambda}{q_i} \sum_{\vec{\boldsymbol{d}} \in \mathcal{D}} p(\vec{\boldsymbol{d}})\, d_i r_i^B(\vec{\boldsymbol{d}})}$$

All that remains: finding $r_i^I(\vec{\boldsymbol{d}})$ and $r_i^B(\vec{\boldsymbol{d}})$

Analyzing ⟨**GEN, CID**⟩: $r_i^I(\vec{\boldsymbol{d}})$

$r_i^I(\vec{\boldsymbol{d}})$: prob. job is sent to the tagged server, given $\vec{\boldsymbol{d}}$ and tagged server is **idle**

Define $b_i(\vec{\boldsymbol{d}})$: probability that all faster queried servers are busy

$$b_i(\vec{\boldsymbol{d}}) = \prod_{j=1}^{i-1} {\rho_j}^{d_j}$$

Job is then assigned to a class-i server w.p. $\alpha_i(i, \vec{\boldsymbol{d}})$

$$\boxed{r_i^I(\vec{\boldsymbol{d}}) = b_i(\vec{\boldsymbol{d}})\alpha_i(i, \vec{\boldsymbol{d}}) \sum_{a_i=1}^{d_i} \binom{d_i - 1}{a_i - 1} \frac{(1-\rho_i)^{a_i - 1} {\rho_i}^{d_i - a_i}}{a_i}}$$

Analyzing ⟨**GEN**, **CID**⟩: Putting it all together

Solve the system of equations:

$$\boxed{\rho_i = \frac{\lambda_i^I}{\mu_i - \lambda_i^B + \lambda_i^I}}$$

$$\boxed{\lambda_i^I = \frac{\lambda}{q_i} \sum_{\vec{\boldsymbol{d}} \in \mathcal{D}} p(\vec{\boldsymbol{d}})\, d_i r_i^I(\vec{\boldsymbol{d}})}$$

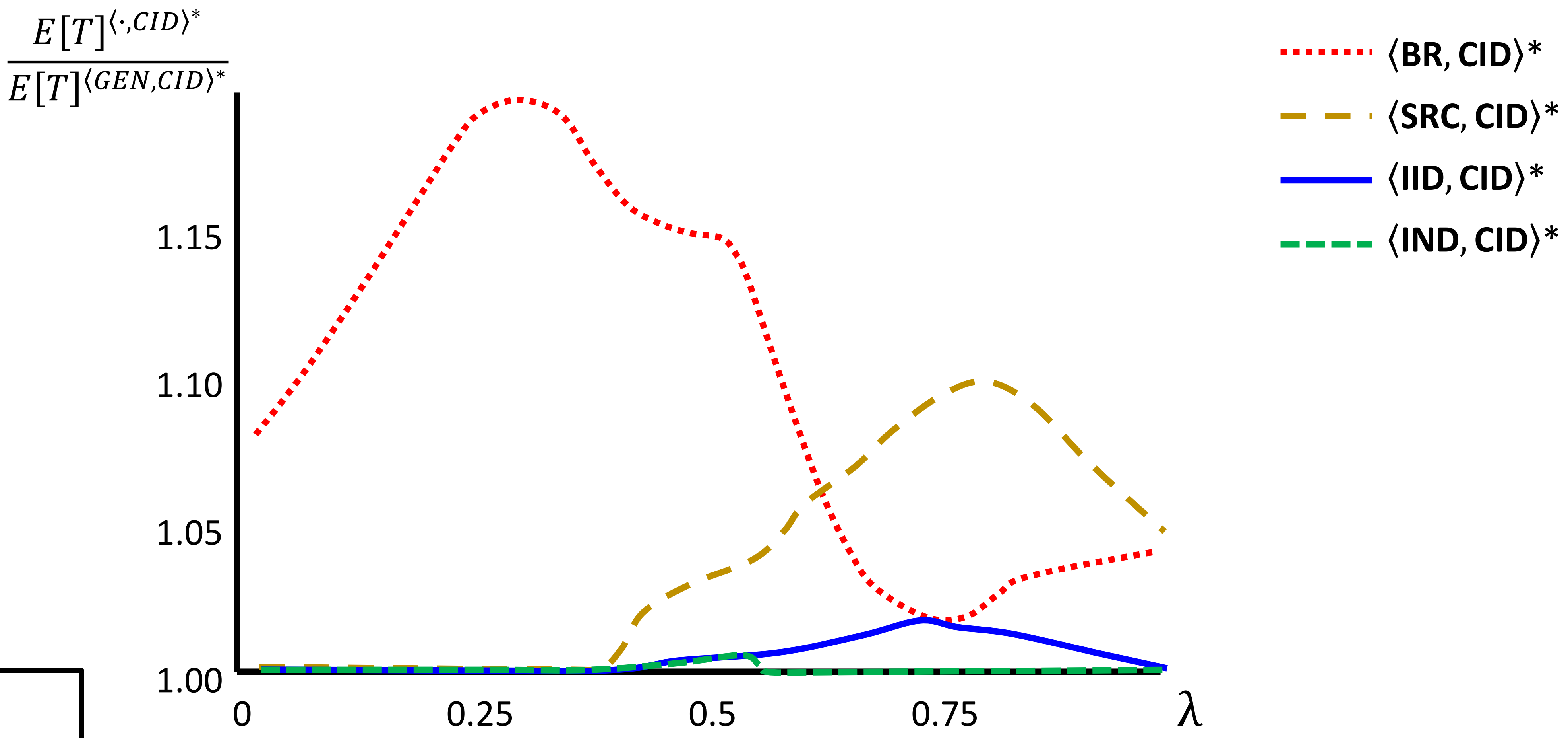
$$\boxed{\lambda_i^B = \frac{\lambda}{q_i} \sum_{\vec{\boldsymbol{d}} \in \mathcal{D}} p(\vec{\boldsymbol{d}})\, d_i r_i^B(\vec{\boldsymbol{d}})}$$

$$\boxed{r_i^I(\vec{\boldsymbol{d}}) = \alpha_i(i, \vec{\boldsymbol{d}}) b_i(\vec{\boldsymbol{d}}) \sum_{a_i=1}^{d_i} \binom{d_i - 1}{a_i - 1} \frac{(1-\rho_i)^{a_i - 1} \rho_i{}^{d_i - a_i}}{a_i}}$$

$$\boxed{r_i^B(\vec{\boldsymbol{d}}) = \frac{\alpha_i(0, \vec{\boldsymbol{d}})}{d_i} \cdot \frac{1}{\rho_i} \prod_{j=1}^{s} \rho_j{}^{d_j} + \sum_{j=i+1}^{s} \frac{\alpha_i(j, \vec{\boldsymbol{d}}) b_j(\vec{\boldsymbol{d}}) (1-\rho_j)^{d_j}}{d_i \rho_i}}$$

$$E[T] = \sum_{i=1}^{s} q_i \left(\frac{(1-\rho_i)\lambda_i^I + \rho_i \lambda_i^B}{\lambda} \right) E[T_i]$$

Numerical Results: $\langle \cdot, \mathbf{CID} \rangle$

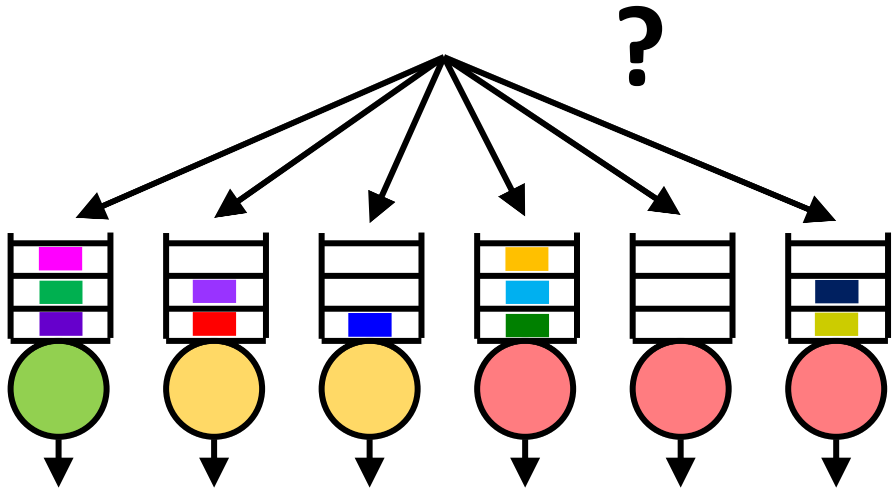


$s = 3$
$d = 3$
$(q_1, q_2, q_3) = \left(\frac{1}{3}, \frac{1}{6}, \frac{1}{2}\right)$
$(\mu_1, \mu_2, \mu_3) = \left(2, \frac{4}{5}, \frac{2}{5}\right)$

Conclusion

Querying Rule

Which d servers to query?



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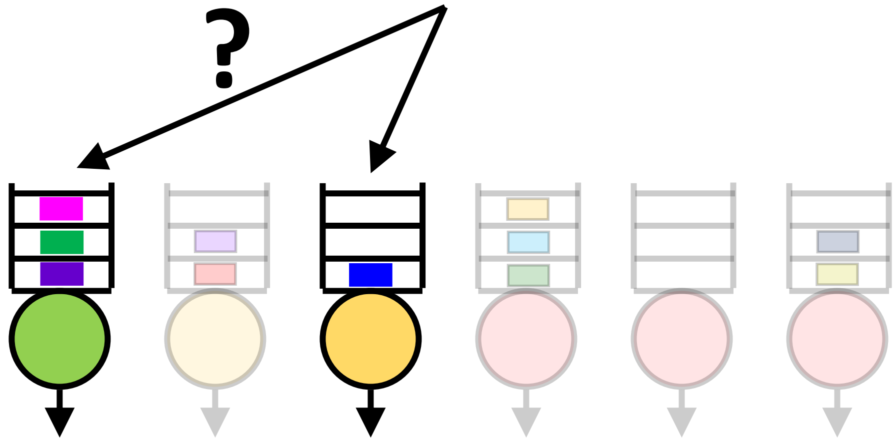

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- Amherst College Computer Science is hiring!

- I'll be visiting the Netherlands for (some part of) the upcoming academic year!

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