

POSTER SESSION, YEP 2023

- Zsuzsana Baran

Phase transition for random walks on graphs with added weighted random matching

Abstract: for a finite graph $G = (V, E)$, we let G^* be the random graph obtained by placing edges of weight ε between pairs of vertices of a random perfect matching. We establish a phase transition in the occurrence of cutoff of a weighted random walk on G^* in terms of the weight ε for graphs of polynomial volume growth and for expanders. Joint work with Jonathan Hermon, Perla Sousi and Andjela Sarkovic.

- Bastien Dubail

Cutoff for mixtures of permuted Markov chains

Abstract: let P_1, P_2 be two stochastic matrices of size n , S a permutation matrix chosen uniformly at random, and $p \in (0, 1)$. We investigate the mixing properties of the Markov chain with transition matrix $pP_1 + (1 - p)SP_2S^{-1}$. With mild assumptions on P_1, P_2 , we prove that with high probability, if its starting point is chosen uniformly at random, then this Markov chain exhibits cutoff at entropic time $\log n/h$, while this may not be true for an arbitrary starting point. Here h is the asymptotic entropy of a lifted version of the Markov chain. In fact, our result is proved for a slightly more general model, allowing to recover the case of the simple random walk on a graph with an added uniform matching proved by Hermon, Sly and Sousi, and extend it to a non-reversible setting.

- John Stewart Fabila-Carrasco.

Dynamics, random graphs and graph signals

Abstract: entropy metrics, such as dispersion entropy, are nonlinear measures of irregularity in time series data defined on path graphs. These entropy metrics can be generalized to data on periodic structures, such as grids or lattice patterns, by using symmetry, enabling their application to images [1]. However, dispersion entropy has not been developed for signals sampled on irregular domains defined by a graph. In this work, we define an entropy metric to analyze signals measured over irregular graphs by generalizing dispersion entropy. Our algorithm compares signal values on neighboring nodes using the adjacency matrix, and we show that this generalization preserves the properties of classical dispersion entropy for time series and recent dispersion entropy for images. It can be applied to any graph structure with synthetic and real signals, including, for example:

(1) MIX processing on random geometric graphs, a family of stochastic processes that overlay sine functions with completely random dynamics using i.i.d. samples selected uniformly.

(2) Small world graph, which are characterized by nodes that are connected to other nodes that are close to them, but also have short paths between nodes that are far apart in the graph. These types of graphs are often used to model social networks.

We conclude with an analysis of the spectrum of the Laplacian [2] and a comparison of

dispersion entropy values to the classical definition of smoothness of a graph signal measured in terms of a quadratic form of the graph Laplacian.

We expect this work to enable the extension of other nonlinear dynamic approaches to graph signals.

References:

1. J. S. Fabila-Carrasco, C. Tan, and J. Escudero, Permutation Entropy for Graph Signals, *IEEE Trans. Signal Inf. Process. over Networks*, 8 (2022) 288–300.
2. J. S. Fabila-Carrasco, F. Lledo, and O. Post, Spectral gaps and discrete magnetic Laplacians, *Linear Algebra Appl.* 547 (2018) 183–216.

- Alicja Kołodziejaska

Weak quenched limit theorems for a random walk in a sparse random environment

Abstract: we consider a perturbed version of the simple symmetric random walk on the set of integers. The random walker moves symmetrically with the exception of some randomly chosen sites where we impose random drift. One can show that if the gaps between the marked sites are i.i.d. and regularly varying with sufficiently small index, then there is no strong quenched limit law for the position of the random walker. Therefore we study the quenched limit laws in the context of weak convergence of random measures. Presented results are the outcome of the joint work with Dariusz Buraczewski and Piotr Dyzewski from University of Wrocław.

- Kiran Kumar

On the spectrum of Linial-Meshulam Complex

Abstract: Linial-Meshulam complex is a random simplicial complex on n vertices with a complete $(d - 1)$ -dimensional skeleton and d -simplices occurring independently with probability p . Linial-Meshulam complex is one of the most studied generalizations of the Erdos–Renyi random graph in higher dimensions.

We study the spectrum of adjacency matrices of the Linial-Meshulam complex when $np \rightarrow \lambda$. We prove the existence of a non-random limiting spectral distribution (LSD) and show that the LSD of signed and unsigned adjacency matrices of Linial-Meshulam complex are reflections of each other. Additionally, we show that the LSD is unsymmetric, unbounded and under the normalization $\frac{1}{\sqrt{\lambda d}}$, converges to semicircle law as $\lambda \rightarrow \infty$.

We also derive the local weak limit of the line graph of the Linial-Meshulam complex and study its consequence on the continuous part of the LSD. This poster is part of a joint work with Kartick Adhikari and Koushik Saha.

- Jakob Maier

Generalizing planted graph alignment to asymmetric sparse Erdős–Rényi graphs

Abstract: we consider the partial graph alignment problem on two correlated sparse Erdős–Rényi graphs with differing edge or node densities. Exploiting that these graphs are locally tree-like, we come to consider a hypothesis testing problem on correlated Galton-Watson trees. To solve this problem, we first give several equivalent conditions for the existence of likelihood-ratio tests with vanishing type-I-error and significant power. A refined analysis establishes sharp information-theoretic testability thresholds, depending explicitly on the model parameters. Finally, we show that these same conditions enable the partial graph alignment algorithm MPAlign to succeed. This paper generalizes two recent papers from Ganassali, Mas soulié, Lelarge, and Semerjian to the asymmetric edge and node density case. This extension allows for greater applicability of the results and resolves a special case of the subgraph isomorphism problem.

Key references:

Ganassali, Luca, Laurent Massoulié, and Marc Lelarge. "Correlation detection in trees for planted graph alignment." 13th Innovations in Theoretical Computer Science Conference (ITCS 2022). Schloss Dagstuhl-Leibniz-Zentrum für Informatik, 2022.

Ganassali, Luca, Laurent Massoulié, and Guilhem Semerjian. "Statistical limits of correlation detection in trees." arXiv preprint arXiv:2209.13723 (2022).

- Nandan Malhotra

Spectra of sparse inhomogeneous Erdős–Rényi Graphs

Abstract: we consider the inhomogeneous Erdős–Rényi Random Graph (IERRG) in the sparse setting, i.e., with the probability of attachments $p_N(i, j)$ tending to 0 asymptotically and the expected degree $Np_N(i, j)$ growing to a finite vertex-dependent function $\lambda f(i, j)$. We consider an appropriately scaled adjacency matrix of this graph, which is a symmetric random matrix with independent Bernoulli entries. Borrowing combinatorial and analytical tools from random matrix theory, we analyse the asymptotic behaviour of this matrix's empirical spectral distribution, which is known as the spectral measure associated with the graph.

- Anas Rahman

Moments of the Antisymmetrised Laguerre Ensemble

Abstract: Let X be a matrix of i.i.d. standard real Gaussian variables and J be the deterministic elementary antisymmetric matrix satisfying $J^T = -J$. We find that the mixed moments $m_{k_1, \dots, k_n} := \langle \text{Tr } A^{k_1} \cdots \text{Tr } A^{k_n} \rangle$ (k_1, \dots, k_n positive integers) of the matrix $A = X^T J X$ representing the antisymmetrised Laguerre ensemble are given by counts of locally orientable bicoloured maps akin to those seen in the standard Laguerre ensemble, but with each term of the count having a well-defined weight in $\{-1, 0, 1\}$. Moreover, the generating functions for these mixed moments are characterised by third order loop equations, escaping the paradigm of classical matrix ensembles, whose moment-generating functions are governed by second order loop equations.

- Andjela Sarkovic
Cutoff for random walk on random graphs with a community structure

Abstract: we consider a variant of the configuration model with an embedded community structure, where every vertex has an internal and an outgoing number of half edges. We pick a uniform matching of the half edges subject to the constraints that internal edges in each community are matched to each other and the proportion of half edges between communities i and j being $Q(i, j)$. We prove that a simple random walk on the resulting graph $G = (V, E)$ exhibits cutoff if and only if the product of the Cheeger constant of Q times $\log |V|$ diverges.

- Alexander Van Werde
Universality-based concentration for sums of dependent random matrices

Abstract: we establish concentration inequalities for sums of dependent random matrices. Our results allow for two types of dependencies: First, we consider a model where the summands are generated by a Markov chain. Second, we consider a model where the summands are deterministic matrices scaled by random coefficients. The leading-order term in our results is sharp and identified by a quantity from free probability theory. We discuss applications related to community detection in block Markov chains as well as the study of adjacency matrices of random graphs with dependent edges.

- Jiyuan Zhang
Non-Haar distributed unitary matrices and their spectral statistics

Abstract: the framework of spherical transforms and Pólya ensembles is of utility in deriving structured analytic results for sums and products of random matrices in a unified way. In this presentation, we will carry over this framework to study products of unitary matrices. Those are not distributed via the Haar measure, but still are drawn from distributions where the eigenvalue and eigenvector statistics factorise. We define cyclic Pólya frequency functions and derive the determinantal point processes of the eigenvalue statistics at fixed matrix dimension. An application is carried out for the Brownian motion on the unitary group, where its spectral statistics and the asymptotics of it are analysed. A dip-ramp-plateau effect arises from the analysis which is attracting recent interest from the viewpoints of many body quantum chaos, and the scrambling of information in black holes.

- Haodong Zhu
The rank of sparse symmetric matrices over arbitrary fields

Abstract: Let F be an arbitrary field and $(G_n, d/n)_n$ be a sequence of sparse weighted Erdős–Rényi random graphs on n vertices with edge probability d/n , whose edge weights are drawn from F according to a joint distribution that is invariant under vertex-relabelling. We show that the normalized rank of the adjacency matrix of $(G_n, d/n)_n$ converges in probability to a constant, and derive the limiting expression. Our result also shows that for this general class of sparse symmetric matrices under consideration, the asymptotics of the normalised rank are independent of the edge weights and even the field, in the sense that the limiting constant for the general case coincides with the one previously result of Bordenave, Lelarge and Salez which is established for adjacency matrices of sparse (non-weighted)

Erdős–Rényi matrices over \mathbb{R} . Our proof, which is purely combinatorial in its nature, is based on an intricate adaptation of the novel perturbation method of Coja-Oghlan, Ergür, Gao, Hetterich and Rolvien to the symmetric setting.