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♣ JOHANNES ALT, Institute for applied mathematics, University of Bonn
Localized phase of the Erdős-Rényi graph
Abstract: we consider the Erdős-Rényi graph $G(N, p)$ on N vertices with edge probability p. In this talk, we show that if $pN/logN$ is sufficiently small, then near the spectral edge of $G(N, p)$ the eigenvectors are localized. That is, that the mass of eigenvector is concentrated around a unique vertex of large degree. In certain regimes, we prove that the corresponding eigenvalues form a Poisson point process.
This is based on joint works with Raphael Ducatez and Antti Knowles.

♣ LUISA ANDREIS, Politecnico di Milano
Rare events in sparse random graphs
Rare events for dense random graphs are well described using the theory of large deviations and graphons. When graphs are sparse the picture is less clear, objects that describe globally the graphs and their limits, as graphons do for dense graphs, have not been defined yet. Nevertheless we do have information on how these graphs look like when explored locally from a vertex and this, under some assumptions, gives also information on global properties. In this talk we will give an overview on what is known on rare events in this regime, focusing in particular on large deviation statements on connected components in inhomogeneous random graphs and on links with coagulation processes. This talk is based on joint works with Wolfgang König (WIAS and TU Berlin), Tejas Iyer, Heide Langhammer, Elena Magnanini and Robert Patterson (WIAS).

♣ JEAN BARBIER, ICTP Trieste
Bayes-optimal limits in structured PCA, and how to reach them
Abstract: we study the paradigmatic spiked matrix model of principal components analysis, where the rank-one signal is corrupted by additive noise. While the noise is typically taken from a Wigner matrix with independent entries, here the potential acting on the eigenvalues has a quadratic plus a quartic component. The quartic term induces strong correlations between the matrix elements, which makes the setting relevant for applications but analytically challenging. Our work provides the first characterization of the Bayes-optimal limits for inference in this model with structured noise. If the signal prior is rotational-invariant, then we show that a spectral estimator is optimal. In contrast, for more general priors, the existing approximate message passing algorithm (AMP) falls short of achieving the information-theoretic limits, and we provide a justification for this sub-optimality. Finally, by generalizing the theory of Thouless-Anderson-Palmer equations, we cure the issue by proposing a novel AMP which matches the theoretical limits. Our information-theoretic analysis is based on the replica method,

a powerful heuristic from statistical mechanics; instead, the novel AMP comes with a rigorous state evolution analysis tracking its performance in the high-dimensional limit. Even if we focus on a specific noise distribution, our methodology can be generalized to a wide class of trace ensembles, at the cost of more involved expressions.

♣ SIMON COSTE, Université Paris Cité / LPSM
Eigenvalues of sparse non-symmetric matrices
Abstract: The eigenvalues of non-hermitian matrices with very few nonzero entries do not exhibit the same behaviour as their hermitian counterpart. In this talk I will expose these differences, then present a few results on the extremal eigenvalues and the log-characteristic polynomial of two models of sparse matrices, namely directed Erdős-Rényi graphs with constant average degree, and sums of random permutations. This is based on various works with Ludovic Stephan, Gaultier Lambert and Yizhe Zhu.

♣ PIERFRANCESCO DIONIGI, Leiden University
Spectral Breaking of Ensemble Equivalence
Abstract: in this talk we will revise the concept of Breaking of Ensemble Equivalence (BEE) and its connections with large deviations theory and random matrix theory. We will see how spectral properties of the adjacency matrix of certain random graph ensembles can be used to detect BEE. We will then discuss further research directions.

♣ LAURE DUMAZ, École Normale supérieure Paris
Some aspects of the Anderson Hamiltonian in 1D
Abstract: in this talk, I will present several results on the Anderson Hamiltonian with white noise potential in dimension 1. This operator formally writes « - Laplacian + white noise ». It arises as the scaling limit of various discrete models and its explicit potential allows for a detailed description of its spectrum. We will discuss localization of its eigenfunctions as well as the behavior of the local statistics of its eigenvalues. Around large energies, we will see that the eigenfunctions are delocalized and follow a universal shape given by the exponential of a Brownian motion plus a drift, a behavior already observed by Rifkind and Virág in tridiagonal matrix models. Based on joint works with Cyril Labbé.

♣ LUCA GANASSALI, EPFL
Aligning graphs via detecting correlation in trees
Abstract: graph alignment refers to recovering the underlying vertex correspondence between two random graphs with correlated edges. This problem can be viewed as an average-case and noisy version of the well-known graph isomorphism problem. For correlated Erdős-Rényi random graphs, we will first give insights on the fundamental limits for the planted formulation of this problem, establishing statistical thresholds for partial recovery. Then, motivated by designing an efficient (polynomial-time) algorithm to recover the underlying alignment in a sparse regime, a message-passing algorithm based on testing correlation in trees is proposed. We study this related correlation detection problem in trees and identify a phase transition in the limit of large degrees for the existence of suitable tests, hence giving insights on the performance of the above method for our initial problem on graphs. Based on joint works with Laurent Massoulié, Marc Lelarge and Guilhem Semerjian.

♣ JOHANNES HEINY, Stockholm University
Eigenvalues of large sample correlation matrices
Many fields of modern sciences are faced with high-dimensional data sets. In this talk, we investigate the spectral properties of a large sample correlation matrix R. Results for the spectral distribution, extreme

eigenvalues and functionals of the eigenvalues of R are presented in both light- and heavy-tailed cases. The findings are applied to independence testing and to the volume of random simplices.

♣ IVAN KRYVEN, Utrecht University
Sequential stub matching for fast sampling from the configuration model
Abstract: for a given graphical degree sequence, there is typically a large number of simple graphs that satisfy it. In this talk we will consider the problem of uniformly sampling graphs from this set. This can also be viewed as generating realisations of the configuration model for a random graph and is a useful procedure if one wants to numerically compute expectations. For example, ensemble-average of the bulk of eigenvalue spectrum. On practice, the most stringent requirement for such algorithms is making sure that the resulting graphs are simple (no multi edges or loops) while maintaining uniform probability of sampling. For instance, fulfilling this requirement when the maximum degree increases with the number of edges requires exponential complexity. We will show that *asymptotical* uniformity for sparse graphs with degree bound $O(m^{1/4})$, with being the number of edges, can be achieved in linear time using sequential construction. That is by non-uniformly placing one edge at a time in the course of m steps while updating the probabilities after each step. We will also formulate several generalisations of this algorithm for directed and coloured random graphs. The talk is partially based on https://arxiv.org/abs/2103.15958

♣ CAMILLE MALE, Université de Bordeaux & CNRS
Freeness over the diagonal of large random graphs and matrices
The theory of "Free Probability", created by Dan Virgil Voiculescu to investigate questions on von Neumann algebras, allows us to understand the spectrum of a random matrix that can be written as a function of several independent random matrices. For about 30 years, developpements of this approach has focused on the study of "nice ensembles" of random matrices, such as Wigner matrices and unitarily invariant ensembles. Therefore the concept of "freeness" plays a central role, analogous to the notion of statistical independence in classical theory, and gives efficient algorithms to compute the spectrum of rational functions in these random matrices. About ten years ago, started the development a new type of method in order to tackle this question for random matrices that are not unitarily invariant, such as matrices whose entries are heavy-tailed, adjacency matrices of sparse random graphs, or more generally random matrices that are invariant by conjugation by permutation matrices (and not all unitary matrices). This gives rise to what is now known as the theory of "Traffic Probability", which comes with a new notion of independence that encodes both freeness and statistical independence. The aim of this presentation is to introduce these new set of methods.

♣ JARON SANDERS, Eindhoven University of Technology
Spectral norm bounds for, and singular value distributions of, block Markov chains
A Block Markov Chain (BMC) is a Markov chain whose state space can be partitioned into a finite number of clusters such that the transition probabilities only depend on the clusters. BMCs thus serve as a model for Markov chains with communities. In this talk, I will share with you some of our latest results on the spectrum of random matrices built from the sample path of a BMC. Specifically, for the centered, empirical frequency matrix of a BMC, we have quantified the asymptotic order of the largest singular value [1] and established limiting laws for the singular value distributions [2]. The former result holds even in sparse regimes when the sample path is relatively short.
References:
[1] Jaron Sanders, Albert Senen-Cerda (2022). Spectral norm bounds for block Markov chain random

matrices. Stochastic Processes and their Applications. https://doi.org/10.1016/j.spa.2022.12.004
[2] Jaron Sanders, Alexander Van Werde (2023). Singular value distribution of dense random matrices with block Markovian dependence. Stochastic Processes and their Applications. https://doi.org/10.1016/j.spa.2023.01.

♣ YIZHE ZHU, University of California Irvine
Non-backtracking spectra of random hypergraphs and community detection
Abstract: the stochastic block model has been one of the most fruitful research topics in community detection and clustering. Recently, community detection on hypergraphs has become an important topic in higher-order network analysis. We consider the detection problem in a sparse random tensor model called the hypergraph stochastic block model (HSBM). We prove that a spectral method based on the non-backtracking operator for hypergraphs works with high probability down to the generalized Kesten-Stigum detection threshold conjectured by Angelini et al (2015). We characterize the spectrum of the non-backtracking operator for sparse random hypergraphs and provide an efficient dimension reduction procedure using the Ihara-Bass formula for hypergraphs. As a result, the community detection problem can be reduced to an eigenvector problem of a non-normal matrix constructed from the adjacency matrix of the hypergraph. Based on joint work with Ludovic Stephan.