"Recent Developments in Stochastic Duality"

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organized by
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Abstracts

Mario Ayala (TU Munich)

Higher-order fluctuation fields and orthogonal duality polynomials

Abstract: In the study of scaling limits of reversible particle systems with the property of self-duality, many quantities of interest become easier to manipulate and simplify. For the particular case of fluctuation limits, and in the additional presence of orthogonality, these simplifications have interesting consequences. In this talk, we will briefly discuss some of those consequences and introduce what we call the k-th order fluctuation field. We will then explain how these fields can be interpreted as some type of discrete analogue of powers of the well-known density fluctuation field, and show how their scaling limits formally correspond to the SPDE associated with the kth-power of a generalized Ornstein-Uhlenbeck process.
This work takes inspiration from [1] and [2], and it is a joint effort with G. Carinci (Università di Modena e R. Emilia) and Frank Redig (TU Delft).

[1] Sigurd Assing, A limit theorem for quadratic fluctuations in symmetric simple exclusion, Stochastic Process. Appl. 117 (2007), no. 6, 766–790.
[2] Patrícia Gonçalves and Milton Jara, Quadratic fluctuations of the symmetric simple exclusion, ALEA Lat. Am. J. Probab. Math. Stat. 16 (2019), no. 1, 605–632.

Guillaume Barraquand (École normale supérieure)

ASEP with boundaries, stationary measures and Markov duality

Abstract: Abstract: In an article published in 1975, T. M. Liggett studied the asymmetric simple exclusion process on a half-line and on a finite domain connected to boundary reservoirs. He assumed that boundary parameters satisfy a specific relation such that reservoirs behave as fictitious sites connected to the boundaries. I will discuss how this assumption allows to:

1. Prove a Markov duality, which leads to explicit formulas for mixed q-moments using coordinate Bethe ansatz (joint work with I. Corwin);
2. Obtain a very explicit description of the non-product stationary measures (joint work with P. Le Doussal).

Jochen Blath (Goethe-Universität Frankfurt)

Dualities in population dynamics and evolution: Recent progress and open problems

Abstract: One of the most prominent and useful instances of a stochastic duality between Markov processes comes from population genetics, namely the classical moment duality between the Wright-Fisher diffusion and the (block counting process of the) Kingman coalescent. Anticipated since the seventies, this duality has a clear genealogical interpretation and has grown into one of the most efficient tools both for empirical and theoretical researchers for the analysis of the interplay of "evolutionary forces". In recent years, several new effects have been described that lead to new types of dualities, including dormancy, switching, coordination and symbiotic branching. In this talk, I review some of the underlying basic models and sketch the effect of the above mechanisms on genealogical relationships. Starting from non-spatial models, I will subsequently move into continuous-space set-ups involving to dualities on the level of (S)PDEs. Along the lines I will sketch several open problems and areas for future research.

Federico Capannoli (Leiden University)

Voter model on directed configuration model

Abstract: We study the classical voter model evolving on the directed configuration model with bounded degree sequence. We are interested in the so-called consensus time, that is, the time it takes the system to reach a monochromatic configuration. Our aim is to find its precise asymptotic behaviour, as the size of the graph grows. We show that in this growing geometry the expected consensus time is linear in the size of the graph, and we identify the exact leading pre-constant in terms of the degree sequence. Our proofs exploit the well-known duality between the voter model and a system of coalescing random walks, plus the mean-field properties of the underlying sparse random environment and its local weak limit. This is an ongoing joint work with Luca Avena, Rajat Subhra Hazra and Matteo Quattropani.

Gioia Carinci (University of Modena and Reggio Emilia)

Condensation of SIP particles and sticky Brownian motion

Abstract: The symmetric inclusion process (SIP) is a particle system with attractive interaction. We study its behavior in the condensation regime attained for large values of the attraction intensity. Using Mosco convergence of Dirichlet forms, we prove convergence to sticky Brownian motion for the distance of two SIP particles. We use this result to obtain, via duality, an explicit scaling for the variance of the density field in this regime, for the SIP initially started from a homogeneous product measure. This provides relevant new information on the coarsening dynamics of condensing particle systems on the infinite lattice. Joint works with M. Ayala, C Giardina and F. Redig

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Benjamin Fehrman (University of Oxford)

Non-equilibrium large deviations and parabolic-hyperbolic PDE with irregular drift

Abstract: In this talk, we will introduce a continuum model that replicates the far-from-equilibrium behavior of certain interacting particle systems. We will show that, along appropriate scaling limits, the solutions to certain stochastic PDEs with conservative noise correctly describe the particle process in terms of a law of large numbers, central limit theorem, and large deviations principle. We will explain the derivation and well-posedness of the SPDE, and we will show that the solutions correctly capture the random fluctuations in the particle system through the detailed analysis of the associated

skeleton equation. This analysis provides the second primary tool of this work, and allows to make rigorous the formal connection above through applications to the identification of l.s.c. envelopes of restricted rate functions, to zero noise large deviations for conservative (singular) SPDE, and to the Gamma-convergence of rate functions.

Chiara Franceschini (University of Modena and Reggio Emilia)

Two duality relations for Markov processes with an open boundary

Abstract: In this talk I will show how the same algebraic approach, which relies on the $su(1,1)$ Lie algebra, can be used to construct two duality results. One is well-known: the two processes involved are the symmetric inclusion process and a Markov diffusion called Brownian Energy process. The other one is a new result which involves a particle system of zero-range type, called harmonic process, and a redistribution model similar to the Kipnis-Marchioro-Presutti model. Despite the similarity, it turns out that the second relation involves integrable models and thus duality can be pushed further. As a consequence, all moments in the stationary nonequilibrium state can be explicitly computed.

Rouven Frassek (University of Modena and Reggio Emilia)

Non-compact spin chains and integrable particle processes

Abstract: I will discuss the relation between non-compact spin chains studied first in the context of high energy physics following ideas of Lipatov, Faddeev and Korchemsky and the zero-range processes introduced by Sasamoto-Wadati, Povolotsky and Barraquand-Corwin. The main difference compared to the prime examples of integrable particle processes, namely the SSEP and the ASEP, is that for the models discussed in this talk several particles can occupy one and the same site. I will introduce integrable boundary conditions for these models that are obtained from the boundary Yang-Baxter equation and which allow to define analogues of the open SSEP and ASEP with boundary reservoirs. Finally, for the rational case with symmetric hopping rates, I will present an explicit mapping of the boundary driven model to equilibrium. This mapping allows to obtain closed-form solutions of the probabilities in steady state and of k-point correlation functions.

Patricia Gonçalves (Instituto Superior Técnico)

Scaling limits for particle exchange models with several conservation laws.

Abstract: In this talk I will present an exclusion process with different types of particles, let us say of type A, B and C. Two scaling limits will be discussed: for the system with an open boundary setting, the hydrodynamic limits; and when evolving on the torus, the fluctuations of some conserved fields. In the later case, we distinguish 3 cases, that depend on the choice of the jump rates, in which we get in the limit either the stochastic Burgers equation or the Ornstein-Uhlenbeck process. These results match with the predictions from non-linear fluctuating hydrodynamics. (Joint with G. Cannizzaro, A. Occelli, R. Misturini.)

Adrián González Casanova (National University of Mexicoé)

Sampling Duality

Abstract: Sampling Duality is stochastic duality using a duality function $S(n, x)$ of the form "what is the probability that all the members of a sample of size n are of type -, given that the number (or frequency) of type - individuals is x". Implicitly this technique can be traced back to the work of Pascal. Explicitly it is studied in a paper of Martin Möhle in 1999. We will discuss several examples in which this technique is useful, including Haldane's formula and the long standing open question of the rate of the Muller Ratchet.

Andreas Greven (University of Erlangen)

The role of duality in genealogy-valued stochastic processes

Abstract: In the literature spatial population models like the interacting Fleming-Viot diffusions or interacting Feller diffusions ,the so-called super random walk,have been studied in great detail and the moment dualities with spatial coalescents played a key role in the analysis.
With these models is naturally associated an evolving genealogical structure, which can be modeled using marked metric measure spaces.Also on this level moment dualities with coalescent genealogies exist and play a very important role.
We focus on the super random walk and explain here the modelling of genealogies and the dualities ,including in addition to the normal duality ,also Feynman-Kac dualities ,strong and conditional ones . In particular this allows to represent the genealogy of the super random walk ,using the configuration of the super random walk together with the genealogy of a specific time inhomogeneous and spatially inhomogeneous Fleming-Viot evolving genealogy.
This is joint work with Andrej Depperschmidt.

Wolter Groenevelt (Delft University of Technology)

A dynamic asymmetric exclusion process with orthogonal dualities

Abstract: In this talk I will discuss dualities of a dynamic asymmetric exclusion process, which generalizes the dynamic asymmetric simple exclusion process by allowing more than one particle on each site. The process can be studied using representation theory of the quantum algebra $U_q(sl_2)$. This leads to several orthogonal dualities that can be described in terms of q-hypergeometric orthogonal polynomials. This is joint work with Carel Wagenaar.

Sabine Jansen (LMU Munich)

Duality, orthogonal polynomials and intertwining for interacting particles in $\mathbb{R}^d$

Abstract: Several interacting particle systems on the lattice have factorized duality functions in which the duality function is a product of single-site duality functions. The latter might be falling factorials or discrete orthogonal polynomials. The talk investigates what happens for interacting particle systems in $\mathbb{R}^d$. Our main results are: (1) Falling factorial dualities generalize as self-intertwining relation with Lenard's K-transform. (2) Reversible consistent conservative particle systems satisfy an self-intertwining relation phrased with generalized orthogonal polynomials as introduced in chaos decompositions and Levy random fields. (3) We introduce a continuum version of the symmetric inclusion process, prove that the law of the negative binomial point process (Pascal point process) is reversible, and deduce self-intertwining relation with generalized Meixner polynomials. Based on joint work with Simone Floreani, Frank Redig and Stefan Wagner (arXiv:2112.11885 [math.PR]).

Jeffrey Kuan (Texas A&M University)

Shift invariance of the multi-species q–TAZRP

Abstract: The Airy_2 process is a universal distribution which describes fluctuations in models in the Kardar-Parisi- Zhang (KPZ) universality class, such as the asymmetric simple exclusion process (ASEP) and the Gaussian Unitary Ensemble (GUE). Despite its ubiquity, there are no proven results for fluctuations of multi–species models. Here, we will discuss one model in the KPZ universality class, the q-Boson. We will show that the joint multi-point fluctuations of the single-species q-Boson match the single-point fluctuations of the multi-species q-Boson. Therefore the single- point fluctuations of multi-species models in the KPZ class ought to be the Airy_2 process.

Jorge Kurchan (École normale supérieure)

Mapping out of equilibrium into equilibrium

Abstract: Years ago it was noted that the Simple Symmetric Exclusion Process and the Kipnis Marchioro Presutti model have the property (in the hydrodynamic limit) that the situation driven by different baths at the edges could be mapped onto a process in contact with an equilibrium bath. This was offered as an explanation for the fact that Bertini, De Sole, Gabrielli, Jona-Lasinio and Landim were able to find an explicit formula for the large deviations. It turns out that the reason for the existence of this mapping is a non-abelian symmetry in the evolution operator, a question unrelated to integrability. Quite surprisingly, the whole discussion may be quite easily extended to the quantum version of the models, just keeping in mind the algebraic structure.

Jan Niklas Latz (Czech Academy of Sciences)

Monoid duality with a special focus on order-theoretic lattices

Abstract: I present an approach to identify pathwise dualities between interacting particle systems with finite local state spaces via commutative monoids (i.e. semigroups containing a neutral element). This approach reveals formerly unknown dualities on local state spaces with more than two elements and, moreover, it allows us to treat several known dualities in a uniform framework. In particular, additive dualities on lattices can be interpreted as special cases of monoid duality. As an example I will revisit the two-stage contact process discovered by Krone and construct its duality from the perspective of monoid duality. This is joint work with Jan Swart.

Christian Maes (KU Leuven)

Matrix-forest theorem for driven and active lattice gases

Abstract: We show how graphical representations help to understand low-temperature nonequilibria, especially for their heat capacity (extended Nernst Postulate).

Reference: arXiv:2210.09858v1 [cond-mat.stat-mech], joint work with Faezeh Khodabandehlou, Irene Maes, Karel Netočný

William Mead (University of Melbourne)

ASEP Dualities from Integrable Vertex Models

Abstract: We demonstrate a method for obtaining expectation values of duality observables in the ASEP. This approach is then mimicked using an integrable vertex model which allows for some generalisations of the duality observables for higher-rank exclusion processes.

Martin Möhle (University of Tübingen)

On some scaling limits for branching processes and regular exchangeable coalescents

Abstract: The first part of the talk provides scaling limits for continuous-time branching processes with discrete state space as the initial state tends to infinity. Depending on the finiteness or non-finiteness of the mean and/or variance of the offspring distribution, the limits are in general time-inhomogeneous Gaussian processes, time-inhomogeneous generalized Ornstein–Uhlenbeck type processes or continuous-state branching processes. We also provide transfer results showing how specific asymptotic relations for the probability generating function of the offspring distribution carry over to those of the one-dimensional distributions of the branching process.
In the second part of the talk we provide scaling limits for the (properly transformed) block counting process of coalescent processes with multiple collisions (Λ-coalescents) restricted to a sample of size n as the initial state n tends to infinity under the assumption that the characterizing measure Λ satisfies $\Lambda([0,x])/x \to \kappa$ as $x \to 0$ for some constant κ. The arising limiting processes belong to the class of Ornstein–Uhlenbeck type processes. Via Siegmund duality a similar convergence result is obtained for the fixation line of the coalescent. The results are extended to a class of regular exchangeable coalescents allowing for simultaneous multiple collisions (Ξ-coalescents). Typical examples are provided.
The talk is based on joint works with Benedict Vetter.

Tomohiro Sasamoto (Tokyo institute of technology)

Current large deviation for the 1D symmetric simple exclusion process

Abstract: We explain the two approaches to study the large deviation for the 1D symmetric simple exclusion process(SEP). The first one is to use the Markov duality of SEP and then use the methods of integrable systems such as the Bethe ansatz[1,2]. This may be considered as a variant of studies on models in the KPZ universality class. The second is to employ the framework of macroscopic fluctuation theory (MFT) and map the problem to a classical integrable systems. This not only reproduces the results of the first approach but also determines the initial and finial density profiles with a given large deviation current[3]. The talk will be based on collaborations with T. Imamura, K. Mallick and H. Moriya.
Refs
[1] T. Imamura, K. Mallick, T. Sasamoto, Large deviations of a tracer in the symmetric exclusion process, PRL 118, 160601 (2017).
[2] T. Imamura, K. Mallick, T. Sasamoto, Distribution of a tagged particle position in the one-dimensional symmetric simple exclusion process with two-sided Bernoulli initial condition, Commun. Math. Phys. 384 (2021) 1409-1444.
[3] K. Mallick, H. Moriya, T. Sasamoto, Exact solution of the macroscopic fluctuation theory for the symmetric exclusion process, PRL 129, 040601 (2022).

Federico Sau (Institute of Science and Technology Austria)

Duality, non-equilibrium and scaling limits

Abstract: In this talk, we discuss a duality-based approach to scaling limits for boundary-driven non-equilibrium systems, with fairly general geometries and reservoirs' intensities. We present some new estimates on moments and correlation functions for the non-equilibrium steady states, which we then apply to derive LLNs and CLTs for the particle systems out of equilibrium.
Based on a joint work with L. Dello Schiavo (ISTA) and L. Portinale (Bonn).

Gunter Schütz (Instituto Superior Técnico)

A reverse duality for the ASEP with open boundaries

Abstract: We prove a duality between the asymmetric simple exclusion process (ASEP) with non-conservative open boundary conditions and an asymmetric exclusion process with particle-dependent hopping rates and conservative reflecting boundaries. This is a reverse duality in the sense that the duality function relates the measures of the dual processes rather than expectations. Specifically, for a certain parameter manifold of the boundary parameters of the open ASEP this duality expresses the time evolution of a family of shock product measures with N microscopic shocks in terms of the time evolution of N particles in the dual process. The reverse duality also elucidates some so far poorly understood properties of the stationary matrix product measures of the open ASEP given by finite-dimensional matrices.

Dario Spanò (University of Warwick)

Pólya urns, duality and eigenstructure for reversible Wright-Fisher-type processes

Abstract: Much of the tractability of neutral, parent-independent Wright-Fisher diffusions of population genetics come from the coexistence of a number of properties: they are polynomial diffusions; they have a moment dual (Kingman's coalescent); they are time-reversible. I will describe the interplay of the above-mentioned properties under a perspective inspired by Bayesian statistics, and will show how this analysis can help in characterising new classes of reversible diffusions, retaining the same stationary measure , whose eigenstructure is driven by a generalised moment dual process.

Rongfeng Sun (National University of Singapore)

A new correlation inequality for Ising models with external fields

Abstract: We study ferromagnetic Ising models on finite graphs with an inhomogeneous external field. We show that the influence of boundary conditions on any given spin is maximised when the external field is identically 0. One corollary is that spin- spin correlation is maximised when the external field vanishes. In particular, the random field Ising model on $\mathbb{Z}^d$, $d \geq 3$, exhibits exponential decay of correlations in the entire high temperature regime of the pure Ising model. Another corollary is that the pure Ising model on $\mathbb{Z}^d$, $d \geq 3$, satisfies the conjectured strong spatial mixing property in the entire high temperature regime. Based on joint work with Jian Ding and Jian Song.

Jan Swart (Czech Academy of Sciences)

Pathwise duality for monotone Markov processes

Abstract: In this talk I will discuss pathwise dualities for interacting particle systems whose local state space is a partially orderd set and whose generators can be represented in terms of monotone

maps. This yields a generalisation of a duality for attractive spin systems due to Gray. In the special case when the local state space is a lattice and the maps are additive, one obtains a generalisation of the classical duality for additive particle systems that also includes other examples such as Siegmund's duality and a duality for the two-stage contact process discoverd by Krone. If the lattice is distributive, we show that each additive duality has an interpretation in terms of a form of oriented percolation. This is joint work with Anja Sturm (Göttingen).

Jon Warren (University of Warwick)

A family of local times studied as a Markov process

Abstract: The classical Ray-Knight Theorem for a Brownian motion B describes the local times of B taken at the first time T_a that B hits a level a : they are distributed as a squared Bessel process of dimension 2. In this talk I will consider a path-valued Markov process $(Z_a; a \geq 0)$ defined by $Z_a(y) = l(T_a, a - y)$ for $y \geq 0$, where $l(.,.)$ are the local times of B. At first sight the evolution of Z seems quite simple: it makes jumps that correspond to the excursions that B makes below its running maximum. But its dual process (in the sense of Markov processes) behaves rather unusually and studying it produces some interesting insights. In particular I will explain a conjecture that if $0 \leq d < 10$, then there exists a random choice of the level a so that $(l(T_a, a-y), 0 \leq y \leq a)$ is distributed not as a squared Bessel process of dimension 2, but as a squared Bessel process of dimension d. On the other hand if $d \geq 10$, then no such random level exists.

Maite Wilke Berenguer (Humboldt Universität Berlin)

Duality and coordination

Abstract: Extensions of the famous Wright-Fisher model typically lead to pairs of (moment-)dual processes, where one is a diffusion and the other a branching-coalescing structure. Using a model for evolutionary selection as an example I will illustrate the idea of "coordinating" such mechanisms resulting in additional jumps to the diffusion and correlation in the branching or coalescing events, respectively, while preserving the moment duality. Given that this can be observed in many models, the question of a description of the general class of such pairs arises.
I will draw from joint work, partially in progress, with Jochen Blath, Maria-Emilia Caballero, Adrian Gonzales Casanova, Nils Hansen, Noemi Kurt, Jose Luis Perez Garmendia, and Dario Spanò.

Anita Winter (Universität Duisburg Essen)

A Feynman Kac duality for structured CSBP with competition under cell division

Abstract: In this talk we introduce a 2-level model which describe cells hosting a virus population. The cells split binary and at a split event divide their virus load to the two daughter cells, while the virus populations within the cells perform independent CSBPs with quadratic competition. We formulate a well-posed martingale problem. An important tool for uniqueness will be a Feynman-Kac duality relation, which can also be used to derive the basic longterm behaviour. We will give criteria for extinction and survival of the virus load within a typical cell sampled at a late time t.
(joint work with Luis Osorio)

Stefan Wagner (LMU Munich)

The algebraic approach to duality in non-discrete spaces

Abstract: The talk develops the Lie algebraic approach to self-duality (or more generally to self-intertwining relations) in the framework of particles evolving on general spaces, e.g. the continuum. In detail, two representations of the current algebra of the Lie algebra su(1,1) are presented. We study the operator that switches between representations to obtain self-intertwiners for consistent and reversible Markov processes, such as the continuum version of the inclusion process. The talk is based on joint work with Simone Floreani (University of Oxford) and Sabine Jansen (LMU Munich).
