NASPDE 2023

EURANDOM

Numerical Analysis of Stochastic Partial Differential Equations

15 – 17 May 2023, Eindhoven

Organisers
Sonja Cox (University of Amsterdam)
Kristin Kirchner (Delft University of Technology)
Gabriel Lord (Radboud University Nijmegen)

Workshop officer
Marianne de Bruin (Eurandom)

Eurandom

Workshop Centre in the area of Stochastics

NASPDE 2023 – Book of Abstracts

Workshop schedule

Monday, 15 May 2023

8:45 – 9:45		Registration
9:45 – 9:50	Organisers	Opening
9:50 – 10:35	Dan Crisan	Noise calibration for geophysical fluid dynamic models
10:35 – 11:00		*Coffee break*
11:00 – 11:45	Svetlana Dubinkina	Projected ensemble data assimilation
11:50 – 12:35	Jonathan Cockayne	Probabilistically quantifying discretisation error in PDE solvers
12:35 – 13:45		*Lunch*
13:45 – 14:30	Annie Millet	Time discretization of the 2D stochastic Boussinesq equation
14:35 – 15:20	Ludovic Goudenège	Tamed Euler scheme for SPDE/SDE with distributional drift
15:20 – 15:50		*Coffee break*
15:50 – 16:35	Aretha Teckentrup	Convergence analysis of non-stationary and deep Gaussian process regression

Tuesday, 16 May 2023

9:00 – 9:45	Claudia Schillings	Subsampling in ensemble Kalman inversion
9:50 – 10:35	Christoph Schwab	Sparsity in UQ for PDEs with Gaussian inputs
10:35 – 11:00		*Coffee break*
11:00 – 11:45	Charles-Edouard Bréhier	A modified Euler scheme for stochastic PDEs
11:50 – 12:35	Mireille Bossy	Stochastic approach to model reduction in computational fluid mechanics. Application to wind time series
12:35 – 13:45		*Lunch*
13:45 – 14:30	Stephanie Sonner	Stabilization by boundary noise
14:35 – 15:20	Chiheb Ben Hammouda	Analysis of numerical smoothing with hierarchical approximations, with applications in probabilities/densities computation and option pricing
15:20 – 15:50		*Coffee break*
15:50 – 16:35	Poster presenters	Poster pitches
16:35 – 17:20		Poster session with drinks
18:30 –		*Workshop dinner at Gezana Restaurant*

Wednesday, 17 May 2023

9:00 – 9:45	Finn Lindgren	Embedding stochastic PDEs in Bayesian spatial statistics software
9:50 – 10:35	Antonis Papapantoleon	Well-posedness and convergence rates for Lévy driven BSDEs
10:35 – 11:00		*Coffee break*
11:00 – 11:45	Konstantinos Dareiotis	Approximation of stochastic PDEs with measurable reaction term
11:50 – 12:35	David Cohen	Analysis of a positivity-preserving splitting scheme for some nonlinear stochastic heat equations
12:35 – 12:40	Organisers	Closing

Speakers, titles and abstracts

Analysis of numerical smoothing with hierarchical approximations, with applications in probabilities/densities computation, and option pricing

Chiheb Ben Hammouda (RWTH Aachen University)

In several applications, when approximating the expectations of a functional of a solution to a stochastic differential equation, the numerical performance and robustness of deterministic quadrature method (e.g., quasi-Monte Carlo (QMC), adaptive sparse grid quadrature (ASGQ)) and multilevel Monte Carlo (MLMC) methods may highly deteriorate owing to the low regularity of the observable with respect to the input parameters. To overcome this issue and improve the regularity structure of the problem, we consider cases in which analytic smoothing (bias-free mollification) cannot be performed, and we introduce in [1] a novel numerical smoothing approach by combining a root-finding method with a one-dimensional numerical integration with respect to a single well-chosen variable. We prove that, under appropriate conditions, the resulting function of the remaining variables is highly smooth, potentially affording the improved efficiency of ASGQ and QMC methods, particularly when combined with hierarchical transformations (i.e., the Brownian bridge and Richardson extrapolation on the weak error). In [2], we extend our idea of numerical smoothing to the MLMC setting. Our analysis and numerical experiments show that the employed numerical smoothing significantly improves the strong convergence of MLMC, and consequently, the complexity and robustness (making the kurtosis at deep levels bounded) of the MLMC method. In particular, we show that we recover the MLMC complexities obtained for Lipschitz functionals when using the Euler-Maruyama scheme. When using a high-order scheme, i.e., the Milstein scheme, we illustrate that we achieve the canonical complexity of MLMC. Our study in [1,2] is motivated by probability computation, pricing options with a discontinuous payoff, and density estimation problems for dynamics where the discretization of the underlying stochastic processes is necessary.

[1] Bayer, C., Ben Hammouda, C., and Tempone, R. "Numerical Smoothing with Hierarchical Adaptive Sparse Grids and Quasi-Monte Carlo Methods for Efficient Option Pricing." Quantitative Finance 23, no. 2 (2023): 209-227.

[2] Bayer, C., Ben Hammouda, C., and Tempone, R. "Multilevel Monte Carlo Combined with Numerical Smoothing for Robust and Efficient Option Pricing and Density Estimation." arXiv preprint arXiv:2003.05708 (2022).

Stochastic approach to model reduction in computational fluid mechanics. Application to wind time series

Mireille Bossy (Inria)

How to conciliate the time series of the wind (at one point) and the fluid mechanics PDEs and their complex range of parameterizations?
By adopting a Lagrangian and stochastic point of view, we construct, from the 3D+time fluid mechanics PDE, a 0D+time stochastic model for the time series of the local variability of the measured wind.
Remarkably, this reduction procedure leads to a well-known stochastic process, the CIR process that allows simple calibration technique. The resulting calibration of the model against wind observation (without, then with uncertainty) is well consistent with the physical parameterisations classically injected in this type of PDE problem, and often resulting from much more controlled laboratory measurements. This result opens perspectives for parameterizing even more sophisticated models (fluid dynamics PDE and times series).

A modified Euler scheme for stochastic PDEs

Charles-Edouard Bréhier (CNRS & Université Lyon 1)

I will present a new temporal discretization scheme for a class of parabolic semilinear stochastic PDEs. Contrary to the standard semi-implicit Euler scheme, the new scheme is shown to preserve the spatial regularity of the exact solution, for any value of the time-step size. The main result for the proposed scheme is an approximation in the sense of the total variation distance of the invariant distribution, when the nonlinearity is a gradient. The talk will focus on the construction of the scheme and on some further applications.

Probabilistically quantifying discretisation error in PDE solvers

Jon Cockayne (University of Southampton)

In this talk we will summarise recent work on probabilistic PDE solvers, highlighting open questions in the area. We will discuss Gaussian-process-based solvers that have a strong relationship to symmetric collocation based methods, and recent work generalising them to apply to the weak formulation of the PDE. We will also discuss the problem of applying this solver to nonlinear PDEs.

Analysis of a positivity-preserving splitting scheme for some nonlinear stochastic heat equations

David Cohen (Chalmers University of Technology)

We construct and analyze a positivity-preserving Lie–Trotter splitting scheme with finite difference discretization in space for approximating the solutions to a class of nonlinear stochastic heat equations with multiplicative space-time white noise.

Noise calibration for geophysical fluid dynamic models

Dan Crisan (Imperial College London)

Stochastic partial differential equations have been used in a variety of contexts to model the evolution of uncertain dynamical systems. In recent years, their applications to geophysical fluid dynamics has increased massively. For a judicious usage in modelling fluid evolution, one needs to calibrate the amplitude of the noise to data. In this paper we address this requirement for the stochastic rotating shallow water (SRSW) model. This work is a continuation of [1], where a data assimilation methodology has been introduced for the SRSW model. The noise used in [1] was introduced as an arbitrary random phase shift in the Fourier space. This is not necessarily consistent with the uncertainty induced by a model reduction procedure. In this paper, we introduce a new method of noise calibration of the SRSW model which is compatible with the model reduction technique. The method is generic and can be applied to arbitrary stochastic parametrizations. It is also agnostic as to the source of data (real or synthetic). It is based on a principal component analysis technique to generate the eigenvectors and the eigenvalues of the covariance matrix of the stochastic parametrization. For SRSW model covered in this paper, we calibrate the noise by using the elevation variable of the model, as this is an observable easily obtainable in practical application, and use synthetic data as input for the calibration procedure. This is joint work with Alexander Lobbe, Oana Lang, Peter Jan van Leewuen and Roland Potthast.

[1] Dan Crisan, Oana Lang, Peter Jan van Leewuen and Roland Potthast. Bayesian inference for fluid dynamics: a case study for the stochastic rotating shallow water model. Frontiers in Applied Mathematics and Statistics, 2023.

Approximation of stochastic PDEs with measurable reaction term

Konstantinos Dareiotis (University of Leeds)

In this talk we will deal with the approximation of stochastic PDEs, in spatial dimension one, of the form

$$\partial_t u = \Delta u + f(u) + \xi, \qquad u(0,x) = u_0(x), \qquad (t,x) \in [0,1] \times \mathbb{T},$$

where ξ is a space-time white noise on $[0,1] \times \mathbb{T}$ and $f: \mathbb{R} \to \mathbb{R}$.
While the approximation of the solution of the above equation has been extensively studied in the case that f is Lipschitz continuous, or at least one-sided Lipschitz, very few results were available for less regular f. In this talk we will show that the rate of convergence of the fully discrete, explicit in time, finite difference scheme is 1/2 in space and 1/4 in time, even for merely bounded, measurable f. The proof relies on the regularisation effect of the noise.
To exploit and quantify this effect we use stochastic sewing techniques.

Projected ensemble data assimilation

Svetlana Dubinkina (Vrije Universiteit Amsterdam)

Data assimilation is broadly used in atmosphere and ocean science to correct model error by periodically incorporating information from measurements (e.g., satellites) into the mathematical model. Both linear and nonlinear data assimilation methods propagate an ensemble of multiple solutions (using different initial conditions with the same numerical model) to approximate the evolution of the probability distribution function (PDF) of plausible states. Linear data assimilation assumes the PDF is Gaussian, while nonlinear data assimilation does not make any assumptions about the PDF. However, the existing nonlinear data-assimilation methods are not used in high-dimensional models as they require a computationally unfeasible ensemble size due to the curse of dimensionality. It is when an ensemble of small size is unable to reduce an error of the estimate. A typical remedy to the curse of dimensionality is distance-based localization. Distance-based localization reduces the model state dimension by taking into account only a few numerical cells of the model state near each observation. Even though distance-based localization reduces the error substantially for both linear data-assimilation methods such as ensemble Kalman filter and nonlinear data-assimilation methods such as particle filtering, linear data-assimilation methods still considerably outperform nonlinear data-assimilation methods in linear and quasi-linear regimes. We propose a further dimension reduction based on projection. We analyze the proposed projected ensemble Kalman filter and the projected particle filter in terms of error propagation. The numerical results show considerable error decrease when used with small ensemble sizes.

Joint work with Jana de Wiljes (University of Potsdam).

Tamed Euler scheme for SPDE/SDE with distributional drift

Ludovic Goudenège (CNRS CentraleSupélec)

Using regularization by noise technics and stochastic sewing lemma from [Le20], we will prove how we can define solutions to SDEs with distributional drift driven by fractional Brownian motion. In the second part, we will extend these technics to SPDE with distributional drift driven by white noise in the spirit of [BDG22]. We build the solutions as the limit of numerical solutions of tamed Euler schemes by combining the time-step convergence and the taming. An optimization in the taming permits obtaining a speed of convergence that recovers the known rates in the bounded regular cases [BDG21, DGL21, DAGI19] but extends the speed of convergence to drifts with negative regularities given constraints on the Hölder regularity of the noise [GHR22].

Joint work with Alexandre Richard (CentraleSupélec, MICS) and El Mehdi Haress (CNRS).

[BDG21] O. Butkovsky, K. Dareiotis, and M. Gerencsér. Approximation of SDEs: a stochastic sewing approach. Probability Theory and Related Fields, 181(4), 975–1034, 2021.

[BDG22] O. Butkovsky, K. Dareiotis, and M. Gerencsér. Optimal rate of convergence for approximations of SPDEs with non-regular drift. arXiv preprint arXiv:2110.06148, 2021.

[GHR22] Ludovic Goudenège, El Mehdi Haress and Alexandre Richard. Numerical approx- imation of SDEs with fractional noise and distributional drift. hal-03715427v1, 2022.

[Le20] K. Lê. A stochastic sewing lemma and applications. Electronic Journal of Prob- ability, 25, 1–55, 2020.

[DGL21] K. Dareiotis, M. Gerencsér and K. Lê. Quantifying a convergence theorem of Gyongy and Krylov. arXiv preprint arXiv:2101.12185, 2021.

[DAGI19] T. De Angelis, M. Germain and E. Issoglio. A numerical scheme for stochastic differential equations with distributional drift. arXiv preprint arXiv:1906.11026, 2019.

Embedding stochastic PDEs in Bayesian spatial statistics software

Finn Lindgren (University of Edinburgh)

The combination of numerical optimisation and integration with finite element constructions of stochastic PDE models have proven to be a versatile and practically useful approach to Bayesian spatial statistics. In order to preserve sparsity of the entire computational pipeline, there are however some practical constraints on the model structures. I will discuss how these aspects are handled in the INLA and inlabru R software packages, including recent developments for non-separable space-time models.

Time discretization of the 2D stochastic Boussinesq equation

Annie Millet (Université Paris 1 Panthéon Sorbonne)

We prove that an implicit time Euler scheme for the 2D-Boussinesq model on the torus D converges. Various moment of the $W^{1,2}$-norms of the velocity and temperature, as well as their discretizations, are computed. We obtain the optimal rate of convergence in probability, and a logarithmic one for the convergence in $L^2(\Omega)$. These results are deduced from a time regularity of the solution both in $L^2(D)$ and $W^{1,2}(D)$, and from an $L^2(\Omega)$ convergence restricted to a subset, where the $W^{1,2}$-norms of the solutions are bounded.

Joint work with Hakima Bessaih.

Well-posedness and convergence rates for Lévy driven BSDEs

Antonis Papapantoleon (TU Delft)

After a brief introduction to BSDEs, we will first provide existence and uniqueness results for BSDEs with jumps driven by martingales that are stochastically discontinuous, hence we can treat BSDEs and BSΔEs in a unified and general framework. Then, we will present stability results for martingale representations and stability results for solutions of BSDEs, not only with respect to the initial data, but also with respect to discretized versions of the driving martingale. Finally, we will discuss some very recent results on convergence rates for BSDEs driven by Lévy processes.

Joint work with Chenguang Liu (Delft), Dylan Possamai (Zürich) and Alexandros Saplaouras (Athens).

Subsampling in ensemble Kalman inversion

Claudia Schillings (FU Berlin)

We consider the ensemble Kalman inversion which has been recently introduced as an efficient, gradient-free optimisation method to estimate unknown parameters in an inverse setting. In the case of large data sets, the ensemble Kalman inversion becomes computationally infeasible as the data misfit needs to be evaluated for each particle in each iteration. Here, randomised algorithms like stochastic gradient descent have been demonstrated to successfully overcome this issue by using only a random subset of the data in each iteration, so-called subsampling techniques. Based on a recent analysis of a continuous-time representation of stochastic gradient methods, we propose, analyse, and apply subsampling-techniques within ensemble Kalman inversion. Indeed, we propose two different subsampling techniques: either every particle observes the same data subset (single subsampling) or every particle observes a different data subset (batch subsampling).

Sparsity in UQ for PDEs with Gaussian inputs

Christoph Schwab (ETH Zürich)

We consider analyticity and sparsity of solution families of PDEs with Gaussian random field (GRF) inputs. Upon parametrizing with suitable affine-parametric representation systems (KL, Parseval frames, ...), such PDEs with GRF input become countably-parametric, deterministic PDEs.
We show the parametric solution families admit analytic continuations into the complex domain, taking values in suitable (corner- and edge-weighted) Sobolev spaces on the polyhedral physical domain. Path regularity of GRF inputs translate into results on sparsity of Wiener polynomial chaos expansions (WPC) of the parametric solutions. This, in turn, implies approximation rate bounds which are free of the curse of dimensionality (CoD) for approximation architectures: reduced basis, sparse grid interpolation, Quasi-Monte Carlo and Smolyak quadratures, deep neural network approximations.
It also implies sparsity results of posteriors in Bayesian PDE inversion subject to uncertain function space input with Gaussian and Besov priors, conditional on observation data with additive, Gaussian observation noise.

Joint work with D. Zung and N. VanKien (Hanoi), Jakob Zech (Heidelberg).

Stabilization by boundary noise

Stefanie Sonner (Radboud University)

The stabilization of parabolic PDEs by multiplicative noise is a well know phenomenon that has been studied extensively over the past decades. However, the stabilizing effect of a noise that acts only on the boundary of a domain had not been investigated so far. As a first model case we consider the Chafee-Infante equation with dynamical boundary conditions and analyze whether a multiplicative Itô noise on the boundary can stabilize the equation. We show that there exists a finite range of noise intensities that stabilize the system. Our results differ from previous works on stabilization, where the noise acts inside the domain and stabilization typically occurs for an infinite range of noise intensities.

Joint work with Klemens Fellner and Bao Q. Tang (University of Graz, Austria) and Do D. Thuan (Hanoi University of Science and Technology, Vietnam).

Convergence analysis of non-stationary and deep Gaussian process regression

Aretha Teckentrup (University of Edinburgh)

We are interested in the task of estimating an unknown function from data, given as a set of point evaluations. In this context, Gaussian process regression is often used as a Bayesian inference procedure, and we are interested in the convergence as the number of data points goes to infinity. Using results from scattered data approximation, we provide a convergence analysis of the method applied to a given, unknown function of interest. We are particularly interested in the case of non-stationary covariance kernels, and the extension of the results to deep Gaussian processes.

Joint work with Conor Osborne (University of Edinburgh).

Poster session

Presenter	Poster title
Pelin Ciloglu	*Adaptive stochastic discontinuous Galerkin methods for PDEs with random data*
El Mehdi Haress	*A tamed-Euler scheme for SDEs with distributional drift and fractional noise*
Katharina Klioba	*Pathwise uniform convergence rates in time for semi-linear SPDEs*
Maximilian Kruse	*Non-parametric Bayesian inference for diffusion processes*
Oana Lang	*A new calibration method for the stochastic shallow water model*
Liam Llamazares Elias	*Penalized complexity priors for stochastic partial differential equations*
Paul Maurer	*Lévy-driven SDEs simulation and application to particles in turbulence*
Khadija Meddouni	*Numerical methods for stochastic neural fields*
Ioanna Motschan	*Simulation of the stochastic heat equation on the sphere*
Jean Daniel Mukam	*Improved estimates for the sharp interface limit of the stochastic Cahn-Hilliard equation with space-time white noise*
Tsiry Randrianasolo	*Wong-Zakai approximation of SPDEs*
Kerstin Schmitz	*Convergence of a finite-volume scheme for a stochastic heat equation with nonlinear multiplicative noise*
Andreas Stein	*Inverse UQ with Besov random tree priors*
Man Ho Suen	*Linearisation approach for aggregated data with INLA-SPDE/inlabru*
Johan Ulander	*Boundary-preserving schemes for a stochastic Allen-Cahn equation*
Joshua Willems	*Markov properties for higher-order parabolic SPDEs*
Fabio Zoccolan	*Dynamical low-rank approximation for stochastic differential equations*

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