# MONDAY 28 AUGUST

## Heavy Loads and Heavy Tails

Speaker: Sem Borst (Eindhoven University of Technology, Netherlands)

Abstract: This talk is concerned with the stationary workload of queues with heavy-tailed (regularly varying) characteristics. We adopt a transform perspective to illuminate a close connection between the tail asymptotics and heavy-traffic limit in infinite-variance scenarios. The aim of the presentation is to provide a tribute to some of the pioneering results of J.W. Cohen in this domain. We specifically demonstrate that reduced-load equivalence properties established for the tail asymptotics of the workload naturally extend to the heavy-traffic limit.

## Heavy-traffic single-server queues and the transform method

Speaker: Johan van Leeuwaarden (Tilburg University, Netherlands)
Joint work with: Marko Boon (Eindhoven University of Technology, Netherlands), Guido Janssen (Eindhoven University of Technology, Netherlands)

Abstract: Heavy-traffic limit theory is concerned with queues that operate close to criticality and face severe queueing times. Let W denote the steady-state waiting time in the GI/G/1 queue. Kingman (1961) showed that W, when appropriately scaled, converges in distribution to an exponential random variable as the system's load approaches 1. The original proof of this famous result uses the transform method. Starting from the Laplace transform of the pdf of W (Pollaczek's contour integral representation), Kingman showed convergence of transforms and hence weak convergence of the involved random variables. We apply and extend this transform method to obtain convergence of moments with error assessment.
The title of this talk and our paper refers to *The Single Server Queue*, the monumental book in which J.W. Cohen teaches the reader how to use complex analysis and transform methods to obtain rigorous results for the general GI/G/1 queue and its many extensions.

## On the cycle maximum of birth-death processes and networks of queues

Speaker: Richard J. Boucherie (University of Twente, Netherlands)

Abstract: We consider the cycle maximum in birth-death processes as a stepping stone to characterisation of the asymptotic behaviour of the maximum number of customers in single queues and open Kelly-Whittle networks of queues. For positive recurrent birth-death processes we show that the sequence of sample maxima is stochastically compact. For transient birth-death processes we show that the sequence of sample maxima conditioned on the maximum being finite is stochastically compact.

We show that the Markov chain recording the total number of customers in a Kelly-Whittle network is a birth-death process with birth and death rates determined by the normalising constants in a

suitably defined sequence of closed networks. Explicit or asymptotic expressions for these normalising constants allow asymptotic evaluation of the birth and death rates, which, in turn, allows characterisation of the cycle maximum in a single busy cycle, and convergence of the sequence of sample maxima for Kelly-Whittle networks of queues.

## Two-dimensional Foreground-Background queueing models

Speaker: Isi Mitrani (University of Newcastle, United Kingdom)
Joint work with: A. Marin (Universita Ca'Foscari Venezia, Italy)

Abstract: Two models involving unbounded foreground and background queues are studied in the steady state. Service is provided either by a single server whose speed depends on the total number of jobs present, or by several parallel servers whose number may be controlled dynamically.
Two different two-dimensional Markov process are solved exactly. Closed-form solutions are derived in some non-trivial special cases.

## Markovian queues with Poisson control

Speaker: Jacques Resing (Eindhoven University of Technology, Netherlands)
Joint work with: Sindo Núñez-Queija (University of Amsterdam, Netherlands), Bala Prabhu (Toulouse University, France)

Abstract: We investigate Markovian queues that are examined by a controller at random times determined by a Poisson process. Upon examination, the controller sets the service speed to be equal to the minimum of the current number of customers in the queue and a certain maximum service speed; this service speed prevails until the next examination time. We study the resulting two-dimensional Markov process of queue length and server speed, in particular two regimes with time scale separation, specifically for infinitely frequent and infinitely long examination times. In the intermediate regime the analysis proves to be extremely challenging. To gain further insight into the model dynamics we then analyse two variants of the model in which the controller is just an observer and does not change the speed of the server.

## A finite compensation procedure for a class of two-dimensional random walks

Speaker: Ioannis Dimitriou (University of Ioannina, Greece)
Joint work with: Ivo Adan (Eindhoven University of Technology, Netherlands)

Abstract: Motivated by queueing applications, we consider a class of two-dimensional random walks, the invariant measure of which can be written as a linear combination of a finite number of product-form terms. In this work, we investigate under which conditions such an elegant solution can be derived by applying a finite compensation procedure. The conditions are formulated in terms of relations among the transition probabilities in the inner area, the boundaries as well as the origin. A discussion on the importance of these conditions is also given.

## The preemptive and non-preemptive combined 'join-the-shortest-queue, serve-the-longest-queue' system with or without switch-over times

Speaker: Uri Yechiali (Tel Aviv University, Israel)
Joint work with: Efrat Perel (Afeka College of Engineering, Israel), Nir Perel (Afeka College of Engineering, Israel)

Abstract: Each of the 2-queue service systems, 'Join-the-Shortest-Queue' (JSQ) and 'Serve-the-Longest-Queue' (SLQ), has been studied separately by J.W. Cohen (1998 and 1987, respectively), and by others. In this work we analyze a combined operating system, denoted JSQ-SLQ, in which arriving customers follow the JSQ policy, while a single server exercises the SLQ discipline. Both preemptive and non-preemptive regimes, with or without switchover times, are investigated. An unconventional system-state formulation with only one infinite dimension is employed, enabling the use of both Probability Generating Functions (PGFs) methodology, and Matrix Geometric analysis.
We derive the joint PGF of the system's states, as well as its stability condition, and calculate system's performance measures such as mean queue sizes, mean waiting times, effective arrival rate to each queue, utilization of each queue, and correlation between the queue sizes. It is shown that the combined JSQ-SLQ system remarkably achieves its goal of balancing the queue sizes. Numerical examples are presented and economic study is carried out, showing the conditions and system parameters under which one regime (preemptive or non-preemptive) is more efficient than the other.

# TUESDAY 29 AUGUST

## On fluctuation-theoretic decompositions via Lindley-type recursions

Speaker: Offer Kella (The Hebrew University of Jerusalem, Israel)
Joint work with: Onno Boxma (Eindhoven University of Technology, Netherlands), Michel Mandjes (Universiteit van Amsterdam, Netherlands)

Abstract: Consider a Lévy process $Y(t)$ over an exponentially distributed time $T_\beta$ with mean $1/\beta$. We study the joint distribution of the running maximum $\bar{Y}(T_\beta)$ and the time epoch $G(T_\beta)$ at which this maximum last occurs. Our main result is a fluctuation-theoretic distributional equality: the vector $(\bar{Y}(T_\beta), G(T_\beta))$ can be written as a sum of two independent vectors, the first one being $(\bar{Y}(T_{\beta+\omega}), G(T_{\beta+\omega}))$ and the second one being the running maximum and corresponding time epoch under the restriction that the Lévy process is only observed at Poisson($\omega$) inspection epochs (until $T_\beta$). We first provide an analytic proof for this remarkable decomposition, and then a more elementary proof that gives insight into the occurrence of the decomposition and into the fact that $\omega$ only appears in the right hand side of the decomposition. The proof technique underlying the more elementary derivation also leads to further generalizations of the decomposition, and to some fundamental insights into a generalization of the well-known Lindley recursion.

## Perishable inventories with random input: a unifying survey with extension

Speaker: David Perry (Holon Institute of Technology, Israel)
Joint work with: Onno Boxma (Eindhoven University of Technology, Netherlands), Wolfgang Stadje (Osnabrück Institute of Mathematics, Germany)

Abstract: This talk is devoted to the theory of perishable inventory systems. In such systems items arrive and stay ' on the shelf' until they are either taken by a demand or become outdated. Our aim is to survey, extend and enrich the probabilistic analysis of a large class of such systems. A unifying principle is to consider the so-called virtual outdating process **V**, where *V(t)* is the time that would pass from *t* until the next outdating if no new demands arrived after *t*. The steady-state density of **V** is determined for a wide range of perishable inventory systems. Key performance measures like the rate of outdatings, the rate of unsatisfied demands and the distribution of the number of items on the shelf are subsequently expressed in that density.

## Analysis of an Emden-Fowler type equation in a Queueing model for EV charging

Speaker: Bert Zwart (Centrum Wiskunde & Informatica CWI and Eindhoven University of Technology)
Joint work: Mark Christianen (University of Twente, Netherlands), Guido Janssen (Eindhoven University of Technology, Netherlands) and Maria Vlasiou (Eindhoven University of Technology, Netherlands)

Abstract: We analyze the properties of a recursive equation that appears as a voltage constraint when scheduling the charging of electric vehicles. We first give an overview of this emerging application area and then show how the recursive equation connects to a specific Emden-Fowler type equation. We derive properties of the continuous solution of this equation and study the asymptotic behavior of its discrete analog.

## Role of Queueing Theory in Telecommunications: Past, Present and Future

Speaker: Hans van den Berg (University of Twente, Netherlands)

Abstract: Developments in telecommunications have been an important driver for research in queueing theory. This started with fundamental questions regarding the dimensioning of telephone networks in the beginning of the 20$^{th}$ century. In this talk we will first present and discuss some highlights of queueing theory applications for the design, management and operations of telecommunication networks, starting from the emergence of the Internet in the 1970's up to current 5$^{th}$ generation cellular mobile networks. However, it's recognized that the role of queueing theory is diminishing. System simulation has always been an important 'competitor', and more recently also AI/ML-based techniques seem to take over the role of queueing theory and other 'traditional' model-based evaluation and optimization methods. We will discuss the potential of

these emerging techniques and also address the question whether queueing theory has really had its day when it comes to applications in telecommunications.

## RBM with Drift in a Wedge

Speaker: Joshua Reed (New York University, USA) ONLINE
Joint work with: Peter Lakner (New York University, USA), Ziran Liu (New York University, USA)

Abstract: We study reflecting Brownian motion with drift constrained to a wedge in the plane. Our first set of results provides necessary and sufficient conditions for existence and uniqueness of a solution to the corresponding submartingale problem with drift. Next, we study a version of the problem with absorption at the vertex of the wedge. In this case, we provide a condition for existence and uniqueness of a solution to the problem and some results on the probability of the vertex being reached. We also prove that in the case of 1 < alpha < 2, RBM in a wedge is a Dirichlet process. Specifically, its unique Doob-Meyer type decomposition is given by Z=X+Y, where X is a two-dimensional Brownian motion and Y is a continuous process of zero energy. Furthermore, we show that for p > alpha , the strong p-variation of the sample paths of Y is finite on compact intervals, and, for 0 < p <= alpha, the strong p-variation of Y is infinite on [0,T] whenever Z has been started from the origin. We also show that on excursion intervals of Z away from the origin, (Z,Y) satisfies the standard Skorokhod problem for X. However, on the entire time horizon (Z,Y) does not satisfy the standard Skorokhod problem for X, but nevertheless we show that it satisfies the extended Skorokhod problem.

## Stationary Brownian motion in a 3/4-plane: Reduction to a Riemann–Hilbert problem via Fourier transforms

Speaker: Guy Fayolle (Inria Paris & Inria Paris-Saclay, France)
Joint work with: Sandro Franceschi (Télécom SudParis & Polytechnique de Paris, France), Kilian Raschel (University of Angers, France)

Abstract: The stationary reflected Brownian motion in a three-quarter plane has been rarely analyzed in the probabilistic literature, in comparison with the quarter plane analogue model. In this context, our main result is to prove that the stationary distribution can indeed be found by solving a boundary value problem of the same kind as the one encountered in the quarter plane, up to various dualities and symmetries. The main idea is to start from Fourier (and not Laplace) transforms, allowing to get a functional equation for a single function of two complex variables.

## Harmonic functions for singular quadrant walks

Speaker: Kilian Raschel (Tours University, France)
Joint work with: Viet Hung Hoang (University of Münster, Germany), Pierre Tarrago (Sorbonne Université, France)

Abstract: We consider discrete random walks confined to the quarter plane, with jumps only in directions (i,j) with i+j≥0 and small negative jumps. These walks are called singular, and were recently intensively studied from a combinatorial point of view. In this paper, we show how the compensation approach introduced in the 90ies by Adan, Wessels and Zijm may be applied to compute positive harmonic functions with Dirichlet boundary conditions. In particular, in case the random walks have a drift with positive coordinates, we derive an explicit formula for the escape probability, which is the probability to tend to infinity without reaching the boundary axes. These formulas typically involve famous recurrent sequences, such as the Fibonacci numbers. As a second step, we propose a probabilistic interpretation of the previously constructed harmonic functions and prove that they allow to compute all positive harmonic functions of these singular walks. To that purpose, we derive the asymptotics of the Green functions in all directions of the quarter plane and use Martin boundary theory.