Abstracts Eurandom workshop:

Random Graphs and Complex Networks

INHOUDSOPGAVE

Abstracts (alphabetical order)

CHRISTIAN BORGS

HERD IMMUNITY AND THE LAW OF LARGE NUMBERS FOR SIR ON THE STOCHASTIC BLOCK MODEL

It is well known that infections spread over contact networks, with contact inhomogeneities playing an important role in the time evolution of the epidemic. In this talk, I will discuss the time evolution of an SIR epidemic on the stochastic block model, and show that both the time evolution over a compact time interval, and the final size of the infection obey a law of large numbers (LLN) described by the solutions of suitable differential equations. As we will see, herd immunity, which is an important concept in epidemiological applications, will also allow us to address certain limitations of standard techniques to prove LLNs, which inherently give errors that explode as time goes to infinity.

This is joint work with my PhD students, Christian Ikeokwu and Karissa Huang.

CHRISTIAN MÖNCH

STRICT INEQUALITIES FOR PERCOLATION THRESHOLDS WITHOUT ESSENTIAL ENHANCEMENTS

Consider translation invariant independent long-range percolation on the d-dimensional lattice with coupling constants F(x,y)=F(0,y-x)=f(y-x), where f is an integrable function. If f' is another coupling function which is coordinate-wise dominated by f, then the percolation threshold for f' is at least as great as that for f. I will discuss a new proof for the fact that this inequality is strict. The argument completely avoids the use of differential inequaities which are needed to implement the classical `essential enhancement'-technique of Aizenmann and Grimmett (1991). Instead, we couple the clusters of the 'smaller' f'-configuration with a *dependent* percolation process on the original model parametrised by f.

As a corollary, we obtain that the critical value for nearest neighbour percolation is strictly decreasing in the dimension, a result due to Kesten (1982). The equivalent statement for the contact process was, perhaps surprisingly, never explicitly proven. This is the main motivation for our approach -- our technique can be transferred without technical effort to contact processes in continuous time.

If time permits, I will also explain how a variation of the construction can be used to show that the critical value of loop percolation is strictly larger than the critical value of nearest neighbour percolation on any Galton-Watson tree with finite branching number larger than 1, and for any ensemble of finite graphs that converges locally to such a tree. These results generalise work of Mühlbacher (2021) for bounded degree graphs.

The talk is based upon joint works in progress with S. A. Bethuelsen (Bergen), and with A. Klippel (Darmstadt) and B. Lees (Leeds).

CLARA STEGEHUIS

DETECTING GEOMETRY IN SCALE FREE NETWORKS

Geometric network models formalize the natural idea that similar vertices are likely to connect. Therefore, geometric models capture many common structural properties of real-world networks. However, if one observes only the network connections, the presence of geometry is not always evident. Currently, triangle counts and clustering coefficients are the standard statistics to signal the presence of geometry. We show that triangle counts or clustering coefficients are insufficient because they fail to detect geometry induced by hyperbolic spaces, or in networks with power law degrees. We, therefore introduce different statistics, based on weighted subgraph counts that can even detect geometry in the 'weak geometry' regime, where the geometric effects converge to zero.

CRISTIAN GIARDINA

SPIN MODELS ON RANDOM GRAPHS

The ferromagnetic Ising spin model is a key statistical mechanics model for continuous phase transitions showing spontaneous magnetization. In this talk, we consider the Ising model on a random graph and show that the additional randomness provided by the graph gives a rich picture with a host of surprises. We identify similarities and differences between the *quenched* and the *annealed* measures obtained by 'fixing' or 'averaging' over the graph realizations. It turns out that the annealed critical temperature is highly model-dependent, even for asymptotically equivalent graphs (such as different versions of the Erdös-Rényi random graph). Instead, the quenched critical temperature is the same for all locally tree-like graphs. Moreover, in the presence of inhomogeneities that produce a fat-tail degree distribution, the difference between quenched and annealed becomes even more substantial, leading to different universality classes and critical exponents. In the last part, we discuss preliminary results on the annealed Potts model, displaying a first-order phase transition.

FRANK DEN HOLLANDER

THE FRIENDSHIP PARADOX FOR SPARSE RANDOM GRAPHS

Consider a group of individuals who form a social network. For each individual in the group, compute the difference between the average number of friends of friends and the number of friends (all friendships are mutual), and average these numbers over all the individuals in the group. It turns out that the latter average is always non-negative, and is strictly positive as soon as not all individuals have exactly the same number of friends. This bias, which at first glance seems counterintuitive, goes under the name of friendship paradox, even though it is a hard fact. In this talk we model the social network as a sparse random graph. We explain where the bias comes from, how it can be quantified, and illustrate it with two examples. We also look at the multi-level friendship paradox, where friends are selected according to an exploration process. We show that different types of scaling can occur as the size of the graph and the depth of the exploration tend to infinity jointly.

Based on joint work with R.S. Hazra and A. Parvaneh

LASSE LESKELÄ

CLUSTERING OF VERTICES IN RANDOM HYPERGRAPHS

Fast algorithms for recovering a latent community structure from an observed hypergraph are usually based on the spectral decomposition of a matrix associated with the hypergraph. Their statistical accuracy can be evaluated by analysing expected error rates for randomly generated hypergraphs with a given community structure, such as the hypergraph stochastic block model (HSBM). Fast and provably exact community recovery algorithms are usually based on aggregating the observed adjacency tensor into a similarity matrix, the entries of which describe the number of hyperedges indicent to a particular node pair.

Unfortunately, the mapping of the adjacency tensor to a similarity matrix loses information, and in some cases this information loss may be statistically significant. Therefore, it still remains unknown whether a fast and exact community recovery algorithm exists in the general setting. Instead of the similarity matrix, an alternative is to apply linear-algebraic techniques to a wide matrix obtained by lossless flattening the adjacency tensor. In this presentation I discuss ongoing research on the analysis of a community recovery algorithm based on flattening the observed hypergraph adjacency tensor. In particular, the approach yields a strongly consistent estimator for recovering communities in d-regular hypergraphs with $n >> 1$ nodes, for which the average hyperedge density is of order at least n^{-d/2}. In still sparser regimes, community recovery is known to be possible but there are no known fast algorithms. These findings suggest that community recovery from d-uniform hypergraphs with $d \geq 3$ may be much harder than what one might expect based on analogous theoretical results for graphs (d=2).

MIA DEIJFEN

GEOMETRIC RANDOM INTERSECTION GRAPHS

An intersection graph is constructed by assigning each vertex a subset of some auxiliary set and then connecting two vertices if their subsets intersect. The model type has been popular in network modeling to describe networks arising from bipartite structures, for instance individuals who are connected if they share a social group, communication units connected via cell towers and scientists related through joint papers. We study a spatial version of the model type where both the vertex set and the auxiliary set are represented by Poisson processes on R^d, giving rise to a variation of the random connection model. Our results concern local quantities (e.g. the degree distribution) and percolation properties of the resulting graph.

MICHEL MANDJES

DYNAMIC RANDOM GRAPHS: ANALYSIS AND INFERENCE

The bulk of the random graph literature concerns models that are of an inherently static nature, in that features of the random graph at a single point in time are considered.

There are strong practical motivations, however, to consider random graphs that are stochastically evolving, so as to model networks' inherent dynamics.
In this talk I'll discuss a set of dynamic random graph mechanisms and their probabilistic properties. Key results cover functional diffusion limits for subgraph counts (describing the behaviour around the mean) and a sample-path large-deviation principle (describing the rare-event behaviour, thus extending the seminal result for the static case developed by Chatterjee and Varadhan).
The last part of my talk will be about estimation of the model parameters from partial information. We for instance demonstrate how the model's underlying parameters can be estimated from just snapshots of the number of edges. We also consider settings in which particles move around on a dynamically evolving random graph, and in which the graph dynamics are inferred from the movements of the particles (i.e., not observing the graph process).

Joint work with Peter Braunsteins, Frank den Hollander, Rajat Hazra, Nikolai Kriukov and Jiesen Wang.

NELLY LITVAK

LOCAL WEAK CONVERGENCE AND HEAVY TAILS OF PAGERANK

PageRank, introduced by Google in 1998 to rank web pages, is one of most common centrality measures in complex networks. In the empirical data, whenever a network, directed or undirected, has a power law (in-)degree distribution, PageRank follows the power law with the same exponent. The so-called power law hypothesis conjectures that this observation holds for all networks with power-law (in-)degree distribution. In this talk I will tell about the exploration of the power law hypothesis in random graph models. An important ingredient of the recent analysis is our result that if a sequence of random graphs converges locally weakly to a rooted random graph, then the PageRank distribution converges to that of the PageRank of the root. While the local weak convergence in itself doesn't say anything about power laws, it does bring us towards resolving the power law hypothesis and yields many unexpected insights such as the striking difference of PageRank properties in directed versus undirected graphs.

PETER MÖRTERS

INHOMOGENEOUS RANDOM GRAPHS WITH KERNEL OF PREFERENTIAL ATTACHMENT TYPE

I will report on the ongoing project on the asymptotic component sizes in the subcritical phase of the inhomogeneous random graph with kernel of preferential attachment type.

PIET VAN MIEGHEM

RANDOM WALKERS INDUCED TEMPORAL GRAPH (RWIG)

We study human mobility networks through timeseries of contacts between individuals, motivated by the COVID pandemic. Nearly all epidemic models assume that the contact graph is static and fixed. We propose the Random Walkers Induced temporal Graph (RWIG) model, which generates temporal graph sequences based on independent random walkers that traverse an underlying graph in discrete time steps. Co-location of walkers at a given node and time defines an individual-level contact. We show that RWIG generates disconnected cliques. We describe the state space induced by RWIG and derive the probability of contact graphs analytically. Finally, a comparison of real temporal networks with contact graphs generated by RWIG justifies RWIG's applicability.

RAJAT HAZRA

LOCAL LIMIT OF PREFERENTIAL ATTACHMENT MODEL AND PERCOLATION

In this talk I will present the local limit of preferential attachment models where vertices enter the network with i.i.d. random numbers of edges that we call the out-degree. We identify the local limit of such models, substantially extending the work of Berger et al.(2014). The degree distribution of this limiting random graph, which we call the random Pólya point tree, has a surprising size-biasing phenomenon. Our models incorporate negative values of the preferential attachment fitness parameter, which allows us to consider preferential attachment models with infinite-variance degrees. We shall use the local limit to identify the percolation phase transition on preferential attachment models. We identify the critical percolation threshold explicitly using the branching process limit.

This is based on joint works with Alessandro Garavaglia, Remco van der Hofstad and Rounak Ray.

STEFFEN DEREICH

TRACES LEFT BY RANDOM WALKS ON LARGE RANDOM GRAPHS

In this talk, we consider traces left by a random walk in the vicinity of a randomly chosen reference vertex. The analysis is related to interlacements but goes beyond previous research by showing weak limit theorems for the vicinity around the reference vertex together with the path segments of the random walk in this vicinity.

Roughly speaking, our limit theorem requires appropriate mixing properties for the random walk together with a stronger variant of Benjamini-Schramm convergence for the underlying graph model. If these assumptions are satisfied, the limiting object can be explicitly given in terms of the Benjamini-Schramm limit of the graph model.

SVANTE JANSON

THE NUMBER OF DESCENDANTS IN A RANDOM DIRECTED ACYCLIC GRAPH

Consider a random directed acyclic graph, obtained by recursively adding vertices, where each new vertex has a fixed outdegree d and the endpoints of the d edges from it are chosen uniformly at random among previously existing vertices. For simplicity we assume d = 2. We study the set of vertices that are descendants of a given vertex n (asymptotically, for large n). The main result is that the number of such vertices, divided by \sqrt n, converges in distribution together with all moments. The limit distribution is, up to a constant factor, a chi distribution χ(4).

The talk is based on a recent paper (Random Structures and Algorithms, 2024).